NATURAL SCIENCE REVIEW 3 200707 (2026)
nsr.jinr.int

Supersymmetry at BLTP: Recent progress

I. L. Buchbinder*[1] and E. A. Ivanov†[1]

[1] *Joint Institute for Nuclear Research, 141980, Dubna, Russia*

Abstract

Ten years ago, in a paper [1], a brief historical survey of the research activity in the Sector "Supersymmetry" at the Bogoliubov Laboratory of Theoretical Physics (BLTP) for more than 50 years of its existence has been given. Here, in commemoration of the 70th jubilee of the Joint Institute for Nuclear Research, we review some recent sound advancements in this area. Specifically, we consider the issues of constructing the superfield quantum effective actions in $6D$, $\mathcal{N} = (1, 0)$ supersymmetry and off-shell unconstrained superfield formulations of $\mathcal{N} = 2$ higher spins. In both cases, the harmonic superspace approach plays the decisive role.

DOI: 10.54546/NaturalSciRev.200707

1. Introduction

The paper [1] was devoted to the brief description of the mainstream scientific activity of Sector No. 2 "Supersymmetry" of BLTP for more than fifty years since its foundation. It was stressed there that the most influential pioneering results and methods which have successfully passed the examination by time were the following: 1) notoph as the first example of antisymmetric gauge field [2]; 2) Ogievetsky theorem [3] and the view of the gravitation theory as a theory of spontaneous breaking, with the graviton as a Goldstone field [4]; 3) the inverse Higgs phenomenon in nonlinear realizations [5]; 4) the complex superfield geometry of $\mathcal{N} = 1$ supergravity [6, 7]; 5) the general relationship between linear and nonlinear realizations of supersymmetry [8, 9]; 6) Grassmann analyticity and harmonic superspace [10–15]. As the future prospective directions of research, in [1] there were mentioned the exploring of the geometry and quantum structure of supersymmetric gauge theories and supergravity in diverse dimensions in the superfield approach, as well as the study of various aspects of supersymmetric and superconformal mechanics models in their intertwining relationships with the higher-dimensional field theories and string theory.

Here we will concentrate on the following two themes, which have received a considerable attention for the last 10 years: the entirely new domain of using the harmonic superspace approach for off-shell description of the higher-spin $4D$, $\mathcal{N} = 2$ supersymmetric theories, as well as the further applications of the quantum harmonic $\mathcal{N} = 2$ formalism for calculations of

*Corresponding author e-mail address: buchbinder@theor.jinr.ru
†Corresponding author e-mail address: eivanov@theor.jinr.ru

Received: 30 March 2026; Revised: 4 June 2026; Accepted: 4 June 2026; Published: 19 June 2026

the effective action of $6D$ supersymmetric gauge theories. We would like to remark in advance that in this short review of our activity for the last decade we as a rule refer to some basic papers on the subject, while the more detailed corpus of references can be found in our original works.

2. $\mathcal{N} = 2$ higher spins from harmonic approach

In this Section, we briefly review our approach to constructing the manifest $4D$, $\mathcal{N} = 2$ supersymmetric higher-spin field theories. The theories are formulated in $\mathcal{N} = 2$ harmonic superspace that provides manifest off shell supersymmetry combined with the explicit gauge invariance. Analytic superfields carrying $\mathcal{N} = 2$ massless supermultiplets of higher spins are described and their interactions with the hypermultiplet are presented. The models constructed were further generalized to involve the superconformal couplings. We also discuss how to define $\mathcal{N} = 2$ higher-spin models on AdS background within the harmonic formalism.

The plan of the Section is as follows. In subsection 2.1, we recall basic notions of $4D$, $\mathcal{N} = 2$ harmonic superspace. In subsections 2.2 and 2.3, the structure of analytic gauge potentials describing off-shell $4D$, $\mathcal{N} = 2$ higher-spin multiplets is explained and their couplings to q^+ hypermultiplets are presented. Subsection 2.4 is devoted to generalization to the superconformal case. In subsections 2.5 and 2.6, the AdS_4 version of harmonic superspace is defined and the simplest examples of models invariant under $4D$, $\mathcal{N} = 2$ AdS supersymmetry are given. Subsection 2.7 contains summary and outlook.

2.1. $4D$ harmonic superspace: why does it matter?

Supersymmetry, despite lacking its experimental confirmation so far, is in the heart of the modern mathematical and quantum physics. It allowed one to construct a lot of new theories with remarkable features: supergravities, superstrings, superbranes, $\mathcal{N} = 4$ super Yang–Mills (the first example of the UV-finite quantum field theory), etc. It also exhibited unexpected relations between them, e.g., “gravity/gauge” duality.

The natural approach to supersymmetric theories is the superfield methods. The natural generalization of Minkowski space x^m to supersymmetry is $\mathcal{N}$ **extended Minkowski superspace**

$$\mathcal{M}^{(4|4\mathcal{N})} = \left(x^m,\ \theta^\alpha_i,\ \bar\theta^{\dot\alpha\, i}\right), \quad i = 1, \ldots, \mathcal{N},$$

where $\theta^\alpha_i, \bar\theta^{\dot\alpha\, i}$ are Grassmann coordinates, $\{\theta, \theta\}, \{\theta, \bar\theta\} = 0$.

Supersymmetric theories are adequately formulated off shell in terms of superfields defined on various superspaces. The basic $4D$, $\mathcal{N} = 1$ superspace is a chiral superspace, with half the original $\mathcal{N} = 1$ Grassmann coordinates. Its $\mathcal{N} = 2$ analog is **harmonic superspace**. Indeed, in four dimensions, the only self-consistent off-shell superfield formalism for both $\mathcal{N} = 2$ and $\mathcal{N} = 3$ theories is known to be the harmonic superspace approach [12, 13, 15].

Harmonic $\mathcal{N} = 2$ superspace is defined as the coordinate set

$$Z = (x^m,\ \theta^\alpha_i,\ \bar\theta^{\dot\alpha\, j}, u^{\pm i}), \quad u^{\pm i} \in SU(2)/U(1),\ u^{+i}u^-_i = 1. \tag{2.1}$$

The basic merit of such an extension is the existence of the analytic harmonic $\mathcal{N} = 2$ superspace involving half the number of Grassmann coordinates:

$$\zeta_A = (x^m_A, \theta^{+\alpha}, \bar\theta^{+\dot\alpha}, u^{\pm i}), \quad \theta^{+\alpha,\dot\alpha} := \theta^{\alpha,\dot\alpha i}u^+_i, \quad x^m_A := x^m - 2i\theta^{(i}\sigma^m\bar\theta^{j)}u^+_i u^-_j\,. \tag{2.2}$$

All basic $\mathcal{N}=2$ superfields are analytic:

$$\underline{\text{SYM}}: \quad V^{++}(\zeta_A); \;\; \underline{\text{matter hypermultiplets}}: \; q^+(\zeta_A),\, \bar{q}^+(\zeta_A);$$
$$\underline{\text{supergravity}}: \quad H^{++m}(\zeta_A),\; H^{++\hat{\alpha}+}(\zeta_A),\; H^{++5}(\zeta_A), \hat{\alpha}=(\alpha,\dot{\alpha}).$$

An instructive example is Abelian $\mathcal{N}=2$ gauge theory,

$$V^{++}(\zeta_A), \quad \delta V^{++} = D^{++}\Lambda(\zeta_A),\; D^{++} = \partial^{++} - 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{\alpha\dot{\alpha}}.$$

In Wess–Zumino gauge, the analytic gauge prepotential V^{++} involves just $8+8$ off-shell degrees of freedom which constitute $\mathcal{N}=2$ vector multiplet:

$$\begin{aligned} V^{++}(\zeta_A) &= (\theta^+)^2\phi + (\bar{\theta}^+)^2\bar{\phi} + 2i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}} + \\ &+ (\bar{\theta}^+)^2\theta^{+\alpha}\psi^i_\alpha u^-_i + (\theta^+)^2\bar{\theta}^+_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2 D^{(ik)}u^-_i u^-_k\,. \end{aligned}$$

The invariant action is written with making use of the non-analytic gauge connection V^{--}:

$$\begin{aligned} &S \sim \int d^{12}Z V^{++}V^{--},\; D^{++}V^{--} - D^{--}V^{++} = 0,\; \delta V^{--} = D^{--}\Lambda, \\ &[D^{++}, D^{--}] = D^0, \quad D^0 V^{\pm\pm} = \pm 2\, V^{\pm\pm}. \end{aligned}$$

2.2. Supersymmetry and higher spins

Supersymmetric higher-spin theories provide a bridge between superstring theory and low-energy (super)gauge theories.

The free massless bosonic and fermionic higher-spin field theories were constructed in [16, 17]. The component approach to $4D$, $\mathcal{N}=1$ supersymmetric free massless higher-spin models was worked out in [18, 19]. The complete off-shell $\mathcal{N}=1$ superfield Lagrangian formulation of $4D$, $\mathcal{N}=2$ free higher spins was given in [20, 21]. At the same time, an off-shell superfield Lagrangian formulation for higher-spin **extended** supersymmetric theories, with all supersymmetries manifest, was unknown for long even for free theories.

This gap was filled in [22]. An off-shell manifestly $\mathcal{N}=2$ supersymmetric unconstrained formulation of $4D$, $\mathcal{N}=2$ super Fronsdal theory for integer spins was constructed in the harmonic superspace approach. The general case with the maximal integer spin $\mathbf{s}$ is spanned by the analytic gauge potentials

$$h^{++\alpha(s-1)\dot{\alpha}(s-1)}(\zeta),\; h^{++\alpha(s-2)\dot{\alpha}(s-2)}(\zeta),\; h^{++\alpha(s-1)\dot{\alpha}(s-2)+}(\zeta),\; h^{++\dot{\alpha}(s-1)\alpha(s-2)+}(\zeta), \tag{2.3}$$

where $\alpha(s) := (\alpha_1 \ldots \alpha_s), \dot{\alpha}(s) := (\dot{\alpha}_1 \ldots \dot{\alpha}_s)$. The relevant gauge transformations can also be defined and shown to leave, in the WZ-like gauge, the physical field multiplet $(\mathbf{s}, \mathbf{s-1/2}, \mathbf{s-1/2}, \mathbf{s-1})$ plus some auxiliary fields. The on-shell spin contents of $\mathcal{N}=2$ higher-spin multiplets can be summarized as[1]

$$\begin{aligned} &\underline{\textit{superspin } 1}: \; 1,\, (1/2)^2,\, (0)^2, \\ &\underline{\textit{superspin } 2}: \; 2,\, (3/2)^2,\, 1, \\ &\underline{\textit{superspin } 3}: \; 3,\, (5/2)^2,\, 2, \\ &\ldots\ldots. \\ &\underline{\textit{superspin } s}: \; s,\, (s-1/2)^2,\, s-1. \end{aligned}$$

[1]The superspin of the given supermultiplet is defined as the largest spin of the component fields.

The manifestly $\mathcal{N}=2$ supersymmetric off-shell cubic couplings of the integer superspin $4D, \mathcal{N}=2$ gauge multiplets to the matter hypermultiplets were further constructed in [23, 24]. Quite recently, the harmonic superspace non-conformal construction was generalized to the case of $\mathcal{N}=2$ superconformal multiplets and their hypermultiplet couplings [25], as well as to the AdS$_4$ case [26][2].

2.3. Hypermultiplet couplings

The construction of interactions in the theory of higher spins is a very important (albeit difficult) task (see, e.g., [28]).

In [23] we have constructed the off-shell manifestly $\mathcal{N}=2$ supersymmetric cubic couplings $(\mathbf{\frac{1}{2}}, \mathbf{\frac{1}{2}}, \mathbf{s})$ of an arbitrary higher integer superspin $\mathbf{s}$ gauge $\mathcal{N}=2$ multiplet to the hypermultiplet matter in $4D, \mathcal{N}=2$ harmonic superspace. In our approach, $\mathcal{N}=2$ supersymmetry of cubic vertices is always manifest and off-shell, in contrast, e.g., to the non-manifest light-cone formulations [29, 30].

The starting point is the $\mathcal{N}=2$ hypermultiplet off-shell free action:

$$S = \int d\zeta^{(-4)}\, \mathcal{L}^{+4}_{\text{free}} = -\int d\zeta^{(-4)}\, \frac{1}{2} q^{+a}\mathcal{D}^{++}q^{+}_{a}\,, a = 1,2. \tag{2.4}$$

The analytic gauge potentials for any spin $\mathbf{s}$ with the correct transformation rules are recovered by proper gauge-covariantization of the harmonic derivative $\mathcal{D}^{++}$. The simplest option is gauging of $U(1)$,

$$\begin{aligned}
&\delta q^{+a} = -\lambda_0 J q^{+a}, \quad J q^{+a} = i(\tau_3)^a_{\ b} q^{+b},\\
&\mathcal{D}^{++} \ \Rightarrow\ \mathcal{D}^{++} + \hat{\mathcal{H}}^{++}_{(1)}, \quad \hat{\mathcal{H}}^{++}_{(1)} = h^{++}J,\\
&\delta_\lambda \hat{\mathcal{H}}^{++}_{(1)} = [\mathcal{D}^{++}, \hat{\Lambda}], \quad \hat{\Lambda} = \lambda J \ \Rightarrow\ \delta_\lambda h^{++} = \mathcal{D}^{++}\lambda.
\end{aligned}$$

In $\mathcal{N}=2$ supergravity, that is for $\mathbf{s}=2$,

$$\begin{aligned}
&S_{(2)} = -\int d\zeta^{(-4)}\, \frac{1}{2} q^{+a}\big(\mathcal{D}^{++} + \mathcal{H}_{(2)}\big)q^{+}_{a}, \quad \delta\mathcal{H}_{(2)} = [\mathcal{D}^{++}, \hat{\Lambda}_{(2)}],\\
&\mathcal{H}_{(2)} = h^{++M}(\zeta)\partial_M,\ \hat{\Lambda}_{(2)} = \lambda^{M}(\zeta)\partial_M,\ M := (\alpha\dot{\beta}, 5, \hat{\mu}+).
\end{aligned} \tag{2.5}$$

For higher $\mathbf{s}$ everything goes analogously. E.g., for $\mathbf{s}=3$,

$$\begin{aligned}
&S_{(3)} = -\int d\zeta^{(-4)}\, \frac{1}{2} q^{+a}\big(\mathcal{D}^{++} + \mathcal{H}_{(3)}J\big)q^{+}_{a},\\
&\delta\mathcal{H}_{(3)} = [\mathcal{D}^{++}, \hat{\Lambda}_{(3)}], \quad \mathcal{H}_{(3)} = h^{++\alpha\dot{\alpha}\,M}(\zeta)\partial_M\partial_{\alpha\dot{\alpha}}, \quad \hat{\Lambda}_{(3)} = \lambda^{\alpha\dot{\alpha}\,M}(\zeta)\partial_M\partial_{\alpha\dot{\alpha}}.
\end{aligned} \tag{2.6}$$

2.4. Superconformal couplings

Free conformal higher-spin actions in $4D$ Minkowski space were pioneered in [31–33]. Since then, a lot of works on (super)conformal higher spins followed (e.g., [34, 35], and many others). (Super)conformal higher-spin theories are considered as a basis for all other types of higher-spin models. Non-conformal ones follow from the superconformal ones through couplings to the **superfield compensators**.

[2] A possible way to extend the whole construction to half-integer superspins was proposed in [27].

In [25], the off-shell $4D$, $\mathcal{N}=2$ higher spins and their hypermultiplet couplings were extended to the superconformal case, once again based on the harmonic Grassmann analyticity principle. Rigid $4D$, $\mathcal{N}=2$ superconformal symmetry plays a crucial role in fixing the structure of the theory.

$4D$, $\mathcal{N}=2$ superconformal algebra (SCA) preserves harmonic analyticity and is a closure of the rigid $\mathcal{N}=2$ supersymmetry and special conformal symmetry

$$\delta_\epsilon \theta^{+\hat{\alpha}} = \epsilon^{\hat{\alpha}i}u^+_i, \quad \delta_\epsilon x^{\alpha\dot{\alpha}} = -4i\left(\epsilon^{\alpha i}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\bar{\epsilon}^{\dot{\alpha}i}\right)u^-_i, \quad \hat{\alpha} = (\alpha, \dot{\alpha}),$$
$$\delta_k \theta^{+\alpha} = x^{\alpha\dot{\beta}}k_{\beta\dot{\beta}}\theta^{\hat{\beta}}, \quad \delta_k x^{\alpha\dot{\alpha}} = x^{\rho\dot{\alpha}}k_{\rho\dot{\rho}}x^{\dot{\rho}\alpha}, \quad \delta_k u^{+i} = (4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}k_{\alpha\dot{\alpha}})u^{-i}.$$

What about the conformal properties of various analytic higher-spin potentials? No problems with the spin **1** potential V^{++}:

$$\delta_{sc}V^{++} = -\hat{\Lambda}_{sc}V^{++}, \quad \hat{\Lambda}_{sc} := \lambda^{\alpha\dot{\alpha}}_{sc}\partial_{\alpha\dot{\alpha}} + \lambda^{\hat{\alpha}+}_{sc}\partial_{\hat{\alpha}+} + \lambda^{++}_{sc}\partial^{--}. \tag{2.7}$$

The cubic vertex $\sim q^{+a}V^{++}Jq^+_a$ is invariant up to total derivative provided that

$$\delta_{sc}q^{+a} = -\hat{\Lambda}_{sc}q^{+a} - \frac{1}{2}\Omega q^{+a}, \quad \Omega := (-1)^{P(M)}\partial_M\lambda^M. \tag{2.8}$$

The situation gets more complicated for $\mathbf{s} \geqslant 2$. Requiring $\mathcal{N}=2$ gauge potentials for $\mathbf{s}=2$ to be closed under $\mathcal{N}=2$ SCA necessarily leads to

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + \kappa_2\hat{\mathcal{H}}^{++}_{(s=2)},$$
$$\hat{\mathcal{H}}^{++}_{(s=2)} := h^{++M}\partial_M = h^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + h^{++\alpha+}\partial^-_\alpha + h^{++\dot{\alpha}+}\partial^-_{\dot{\alpha}} + h^{(+4)}\partial^{--}$$
$$\delta_{k_{\alpha\dot{\alpha}}}h^{(+4)} = -\hat{\Lambda}h^{(+4)} + 4ih^{++\alpha+}\bar{\theta}^{+\dot{\alpha}}k_{\alpha\dot{\alpha}} + 4i\theta^{+\alpha}h^{++\dot{\alpha}+}k_{\alpha\dot{\alpha}}.$$

For ensuring conformal covariance, it is impossible to avoid introducing the extra potential $h^{(+4)}$. This extended set of potentials embodies $\mathcal{N}=2$ **Weyl multiplet** ($\mathcal{N}=2$ conformal SG gauge multiplet).

For $\mathbf{s} \geqslant 3$ the gauge-covariantization of the free q^{+a} action requires adding the gauge superfields carried by differential operators of rank $\mathbf{s-1}$ in ∂_M,

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + \kappa_s\hat{\mathcal{H}}^{++}_{(s)}(J)^{P(s)}, \quad P(s) = \frac{1+(-1)^{s-1}}{2}.$$

In particular, for $\mathbf{s}=3$:

$$\hat{\mathcal{H}}_{(s=3)} = h^{++MN}\partial_N\partial_M + h^{++}, \quad h^{++MN} = (-1)^{P(M)P(N)}h^{++NM}. \tag{2.9}$$

The whole consideration can be repeated for the general integer higher-spin s case. The relevant generalized Weyl multiplets involve $8(2s-1)_B + 8(2s-1)_F$ d.o.f. off shell.

The self-consistent $\mathcal{N}=2$ superconformal couplings of higher-spin gauge superfields to the matter q^+ hypermultiplets at the full nonlinear level and for an arbitrary $\mathcal{N}=2$ conformal supergravity background were also presented for the first time in [25]. It is worth noting that the higher-spin $\mathcal{N}=2$ gauge supermultiplets in the description through the Mezincescu-type prepotentials for the particular case of conformally flat backgrounds were earlier considered in [36].

2.5. Towards AdS background

In view of the celebrated AdS/CFT correspondence, it is of high importance to explicitly construct $\mathcal{N}=2$ higher spins in the AdS background, with the superconformal symmetry $SU(2,2|2)$ being broken to the AdS supersymmetry $OSp(2|4;R)$.

One way to achieve this is to start from the covariant formalism in the AdS_4, $\mathcal{N}=2$ superspace defined as the coset $OSp(2|4;R)/[SO(2)\times SL(2,C)]$, thus generalizing the AdS_4, $\mathcal{N}=1$ superfield approach of [37]. This way was chosen in [38], essentially based on the so-called projective superspace techniques and treating the case of $5D$, $\mathcal{N}=1$ AdS superspace (see also a recent paper [39]).

The approach of [26] is distinguished in that it exclusively proceeds from the analyticity-preserving realization of the superconformal symmetry in $\mathcal{N}=2$ harmonic superspace and identifies $\mathcal{N}=2$ AdS supersymmetry $OSp(2|4;R)$ as its subalgebra, $OSp(2|4;R)\subset SU(2,2|2)$. So the super AdS supersymmetry is already implied by the superconformal symmetry. Once again, the manifest $\mathcal{N}=2$ harmonic Grassmann analyticity plays the decisive role.

The embedding of $\mathcal{N}=2$ AdS superalgebra into $SU(2,2|2)$ is realized through the identification [40, 41]

$$\Psi^i_\alpha = Q^i_\alpha + c^{ik}S_{k\alpha}, \quad \bar\Psi^i_{\dot\alpha} = \overline{\Psi^i_\alpha} = \bar Q_{\dot\alpha i} + c_{ik}\bar S^k_{\dot\alpha}, \tag{2.10}$$
$$c^{ik} = c^{ki} \qquad \overline{c^{ik}} = c_{ik} = \varepsilon_{il}\varepsilon_{kj}c^{lj}. \tag{2.11}$$

The $SU(2,2|2)$ commutation relations imply for super AdS generators

$$\begin{aligned}
&\{\Psi^i_\alpha, \Psi^k_\beta\} = -4c^{ik}L_{(\alpha\beta)} + 4i\varepsilon_{\alpha\beta}\varepsilon^{ik}I, \quad I := c_{lm}T^{lm}, \quad [I, \Psi^i_\alpha] = c^{ik}\Psi_{k\alpha},\\
&\{\Psi^i_\alpha, \bar\Psi_{\dot\beta k}\} = 4\delta^i_k\, R_{\alpha\dot\beta}, \quad R_{\alpha\dot\beta} = P_{\alpha\dot\beta} + \frac{1}{2}c^2\, K_{\alpha\dot\beta}, \quad c^2 := c^{ik}c_{ik} \sim \frac{1}{R^2_{AdS}},\\
&[R_{\alpha\dot\alpha}, R_{\gamma\dot\gamma}] = \frac{1}{2}c^2\big(\varepsilon_{\alpha\gamma}\bar L_{\dot\alpha\dot\gamma} + \varepsilon_{\dot\alpha\dot\gamma}L_{\alpha\gamma}\big), \quad [R_{\alpha\dot\beta}, \Psi^i_\beta] = \varepsilon_{\alpha\beta}\,\bar\Psi^i_{\dot\beta}\ \ (\text{and c.c.}).
\end{aligned}\tag{2.12}$$

The **super AdS** transformation can be then read off as

$$\begin{aligned}
&\delta_\epsilon x^{\alpha\dot\alpha} = -4i\left[\epsilon^{\alpha i}\bar\theta^{+\dot\alpha} + \theta^{+\alpha}\bar\epsilon^{\dot\alpha i} - c^{ik}\big(x^{\alpha\dot\beta}\bar\epsilon_{\dot\beta k}\bar\theta^{+\dot\alpha} + x^{\beta\dot\alpha}\epsilon_{\beta k}\theta^{+\alpha}\big)\right]u^-_i,\\
&\delta_\epsilon\theta^{+\alpha} = \big(\epsilon^{\alpha i} - x^{\alpha\dot\alpha}c^{ik}\bar\epsilon_{\dot\alpha k}\big)u^+_i - 2i(\theta^+)^2c^{ki}\epsilon^\alpha_k u^-_i,\\
&\delta_\epsilon\bar\theta^{+\dot\alpha} = \big(\bar\epsilon^{\dot\alpha i} + x^{\alpha\dot\alpha}c^{ik}\epsilon_{\alpha k}\big)u^+_i + 2i(\bar\theta^+)^2c^{ik}\bar\epsilon^{\dot\alpha}_k u^-_i,\\
&\delta_\epsilon u^{+i} = -4i\big[c^{kl}u^+_k(\epsilon_{\alpha l}\theta^{+\alpha} + \bar\epsilon_{\dot\alpha l}\bar\theta^{+\dot\alpha})\big]u^{-i}.
\end{aligned}\tag{2.13}$$

The **nonlinear AdS** translations look as

$$\begin{aligned}
&\delta_a x^{\alpha\dot\alpha} = a^{\alpha\dot\alpha} + \frac{1}{2}c^2 a_{\beta\dot\beta}x^{\alpha\dot\beta}x^{\beta\dot\alpha} = a^{\alpha\dot\alpha}\big(1 - \frac{1}{4}c^2x^2\big) + \frac{1}{2}c^2(ax)x^{\alpha\dot\alpha},\\
&\delta_a\theta^{+\alpha} = \frac{1}{2}c^2a_{\beta\dot\beta}x^{\alpha\dot\beta}\theta^{+\beta} = \frac{1}{4}c^2(ax)\theta^{+\alpha} + \frac{1}{2}c^2x^{(\alpha\dot\beta}a_{\beta)\dot\beta}\theta^{+\beta},\\
&\delta_a\bar\theta^{+\dot\alpha} = \frac{1}{2}c^2a_{\beta\dot\beta}x^{\beta\dot\alpha}\bar\theta^{+\dot\beta} = \frac{1}{4}c^2(ax)\bar\theta^{+\dot\alpha} + \frac{1}{2}c^2x^{\beta(\dot\alpha}a_{\beta\dot\beta)}\bar\theta^{+\dot\beta},\\
&\delta_a u^{+i} = 2i\left(c^2a_{\alpha\dot\alpha}\theta^{+\alpha}\bar\theta^{+\dot\alpha}\right)u^{-i}.
\end{aligned}\tag{2.14}$$

The AdS supersymmetry transformation of the analytic integration measure reads

$$\delta_\epsilon d\zeta^{(-4)} = 8i\left(c^{kl}u^-_k\epsilon_{\alpha l}\theta^{+\alpha}\right)d\zeta^{(-4)}.$$

To compensate this variation of the integration measure in the action of q^{+a}, the latter should include the appropriate weight factor in its transformation,

$$\delta_\epsilon q^{+a} = -4i\left(c^{kl}u^-_k\epsilon_{\alpha l}\theta^{+\alpha}\right)q^{+a}. \tag{2.15}$$

The harmonic derivative $\mathcal{D}^{++}$ transforms under AdS supersymmetry as

$$\delta_\epsilon D^{++} = 4i\,c^{kl}u^+_k\epsilon_{\alpha l}\theta^{+\alpha}\,D^0,\quad D^0q^{+a}=q^{+a},\ q^{+a}q^+_a=0.$$

Then the free q^+ action,

$$S_{(2)} = -\int d\zeta^{(-4)}\,\frac{1}{2}q^{+a}\mathcal{D}^{++}q^+_a,$$

is invariant under the $\mathcal{N}=2$ superconformal transformations and hence under the $\mathcal{N}=2$ super AdS ones. The question is how to construct the actions invariant only under $OSp(2|4;R)$ symmetry. To accomplish this, one needs to properly break $SU(2,2|2)$.

2.6. Two ways of constructing $OSp(2|4;R)$ invariants

We start with the $\mathcal{N}=2$ AdS invariant mass term. The basic idea is to extend the coordinate action of generators of AdS supersymmetry by extra "matrix" pieces of an external $SO(2)$ realized on q^{+a}. In the flat limit such a modified $\mathcal{N}=2$ AdS contracts into the central-charge extended $4D$, $\mathcal{N}=2$ supersymmetry.

Denoting formally this extra $SO(2)$ generator as ∂_5 and introducing

$$q^+(\zeta,x^5)=e^{-im_qx^5}q^+(\zeta),\quad \tilde q^+(\zeta,x^5)=e^{im_qx^5}\tilde q^+(\zeta),\quad [m_q]=l^{-1}, \tag{2.16}$$

find the realization of $OSp(2|4;R)$ by the following Killing super-vector:

$$\begin{aligned}
&\hat\Lambda_{AdS} = \lambda^{\alpha\dot\alpha}_{AdS}\partial_{\alpha\dot\alpha}+\lambda^{\hat\alpha+}_{AdS}\partial^-_{\hat\alpha}+\lambda^{++}_{AdS}\partial^{--}+\lambda^5_{AdS}\partial_5,\quad \lambda^5_{AdS}=\lambda^5_\epsilon+\lambda^5_{so(2)},\\
&\lambda^5_\epsilon = 2ie(x)\Big[(\theta^+\epsilon^-)-x^{\alpha\dot\alpha}\bar\theta^+_{\dot\alpha}(\epsilon^+_\alpha c^{--}-\epsilon^-_\alpha y)\Big]\times\\
&\qquad\times\Big\{1-2i\left[(\theta^+)^2-(\bar\theta^+)^2\right]e(x)c^{--}\Big\}+(c.c.),\\
&\lambda^5_{so(2)} = \frac{\gamma}{2}+i\left[(\theta^+)^2-(\bar\theta^+)^2\right]\gamma c^{--}e(x)-6(\theta^+)^4\gamma\left(c^{--}\right)^2e(x)^2,
\end{aligned} \tag{2.17}$$

where $y=c^{ij}u^+_iu^-_j\,, e(x)=\frac{1}{1+m^2x^2/2}$.

In order to construct an invariant q^+ action, one needs to lengthen $\mathcal{D}^{++}$ as

$$\begin{aligned}
&\mathcal{D}^{++}\ \Rightarrow\ \mathcal{D}^{++}+H^{++5}_{AdS}\partial_5,\\
&H^{++5}_{AdS}=i\left[(\theta^+)^2-(\bar\theta^+)^2\right]e(x)-6(\theta^+)^4e(x)^2\,c^{--},\quad \delta_{AdS}H^{++5}_{AdS}=\mathcal{D}^{++}\lambda^5_{AdS}.
\end{aligned}$$

After elimination of an infinite tail of the auxiliary fields and integrating over Grassmann and harmonic variables, the bosonic part of the massive AdS hypermultiplet action can be written as

$$\begin{aligned}
S_{\rm scalar} &= -\frac{1}{2}\int d\zeta^{(-4)}\,q^{+a}\left(\mathcal{D}^{++}+H^{++5}_{AdS}\partial_5\right)q^+_a\,|_{\rm bos}=\\
&=\int d^4x\,\Big(\partial_nf^i\partial^n\bar f_i-m_q^2e(x)^2f^i\bar f_i-im_qe(x)^2f^i\bar f^jc_{(ij)}\Big),
\end{aligned} \tag{2.18}$$

where we observe the presence of the explicit $SU(2)$ breaking part in the mass term, with the residual $SO(2)$ invariance.

After rescaling $f^i \to e(x)\hat{f}^i$ we obtain the AdS covariant scalar field

$$\delta_a \hat{f}^i(x) = 0 \tag{2.19}$$

and gain the standard kinetic action for the massive scalars given on AdS metric $g_{mn} = e(x)^2\eta_{mn}$ (with the scalar curvature $R = 4\Lambda = -48c^{ik}c_{ik}$):

$$S_{\text{scalar}} = \int d^4x\sqrt{-g}\left(g^{mn}\partial_m\hat{f}^i\partial_n\bar{\hat{f}}_i + \frac{R}{6}\hat{f}^i\bar{\hat{f}}_i - m_q^2\hat{f}^i\bar{\hat{f}}_i - im_q\hat{f}^i\bar{\hat{f}}^j c_{(ij)}\right). \tag{2.20}$$

After passing to the special frame $c_{ij} = m\delta_{ij}$, diagonalizing mass matrix and rewriting $\hat{f}^i = (f_+, f_-)$, one obtains the splitting of mass for the scalar fields proportional to the parameter m,

$$m_\pm^2 = m_q^2 \pm mm_q.$$

For the fermionic fields one needs a similar redefinition, $\psi_\alpha(x) \to e(x)^{\frac{3}{2}}\hat{\psi}_\alpha(x)$. Using the AdS covariant derivative $\nabla_{\alpha\dot{\alpha}} = \left(1 + \frac{m^2}{2}x^2\right)\partial_{\alpha\dot{\alpha}} = e(x)^{-1}\partial_{\alpha\dot{\alpha}}$, one comes to the action for Dirac spinor on AdS space:

$$\begin{aligned} S_{\text{fer}} = \int d^4x\sqrt{-g}\Big\{ & i\bar{\hat{\psi}}^{\dot{\alpha}}\left(\nabla^\alpha_{\dot{\alpha}} - \frac{3}{2}m^2x^\alpha_{\dot{\alpha}}\right)\hat{\psi}_\alpha + i\bar{\hat{\kappa}}^{\dot{\alpha}}\left(\nabla^\alpha_{\dot{\alpha}} - \frac{3}{2}m^2x^\alpha_{\dot{\alpha}}\right)\hat{\kappa}_\alpha + \\ & + \frac{m_q}{2}\left(\hat{\psi}^\alpha\hat{\kappa}_\alpha + \bar{\hat{\psi}}_{\dot{\alpha}}\bar{\hat{\kappa}}^{\dot{\alpha}}\right)\Big\}. \end{aligned} \tag{2.21}$$

So, $m_f = m_q, \quad m_+^2 + m_-^2 = 2m_f^2$, like in $\mathcal{N} = 1$ AdS case [37].

In order to produce a more general class of $\mathcal{N} = 2$ AdS invariant systems one needs to redefine the superfield q^{+a} in such a way that it transforms as a scalar superfield with zero weight under the AdS subgroup of $\mathcal{N} = 2$ superconformal group. This is the superfield Weyl-type transformation preserving $\mathcal{N} = 2$ harmonic analyticity.

We start from the free q^+ action, $S_{\text{free}} = -\frac{1}{2}\int d\zeta^{(-4)}q^{+a}\mathcal{D}^{++}q^+_a$. It is superconformally invariant and hence is invariant under super $\mathcal{N} = 2$ super AdS_4 group. Then one makes a Weyl-type rescaling of q^+,

$$q^{+a} = G^{\frac{1}{2}}\,\hat{q}^{+a},\; G = \frac{\left(1 + \frac{y^2}{m^2}\right)}{\left(1 + \frac{m^2x^2}{2}\right)^2}\left(1 + \theta\,\text{terms}\right),\; y := c^{+-} = c^{ik}u^+_iu^-_k\,, \tag{2.22}$$

so that $\hat{q}^{+a}$ is a scalar of zero weight under $\mathcal{N} = 2$ super AdS_4 group. The $\hat{q}^+$ action takes the form manifestly invariant under this group:

$$S_{\text{free}} = -\frac{1}{2}\int d\zeta^{(-4)}\,G\,\hat{q}^{+a}\mathcal{D}^{++}\hat{q}^+_a\,, \quad \delta_{AdS}\hat{q}^{+a} = 0.$$

The new integration measure $d\zeta^{(-4)}G$ is invariant under $OSp(2|4;R)$. So one can add to the Lagrangian any proper function of $\hat{q}^{+a}$ without breaking of $OSp(2|4;R)$. In particular, one can add an arbitrary $\mathcal{L}^{+4}(\hat{q}^{+a}, u^-)$ and so gain a wide class of the hyper-Kähler type sigma-model actions on the AdS_4 background.

To be more precise, one can consider the following action:

$$S_{HK}^{AdS} = \int d\zeta^{(-4)}\Sigma\,\left[\hat{q}_a^{+}\mathcal{D}^{++}\hat{q}^{+a} + L^{(+4)}(\hat{q}^{+}, w^{+}, u^{-})\right], \tag{2.23}$$

where

$$w^{+i} := u^{+i} - u^{-i}c^{++}\frac{y}{y^2+m^2}, \qquad \delta_{AdS}w^{+i} = 0,$$

is a new harmonic variable which is inert under $OSp(2|4)$. In the properly defined flat limit this action goes over just to the general action of hyper-Kähler $\mathcal{N}=2$ sigma models [11]. It generically does not exhibit any external isometry (besides the intrinsic $SO(2)$), quite similar to its flat $\mathcal{N}=2$ counterparts. It would be interesting to find out the relevant deformation of the hyper-Kähler geometry and to clarify the role of the breaking parameter c^{ik} in it.

2.7. Summary and outlook

The theory of $\mathcal{N}=2$ supersymmetric higher spins $s \geqslant 3$ opened a new promising direction of applications of the harmonic superspace approach which earlier proved to be indispensable for description of more conventional $\mathcal{N}=2$ theories with maximal spins $s \leqslant 2$. Once again, the basic property underlying these new higher-spin theories **is the harmonic Grassmann analyticity** (all basic gauge potentials are unconstrained analytic superfields involving an infinite number of degrees of freedom off shell before fixing WZ-type gauges). As we saw on the example of hypermultiplet, harmonic superspace admits a nice generalization to the case of AdS background where **the harmonic Grassmann analyticity principle** again plays the decisive role. We expect that the same principle will efficiently work as well for various $\mathcal{N}=2$ AdS supergravities and their higher-spin analogs, and this will allow us to get further insights into the geometric and quantum structure of these notable theories. One of the most interesting problems here is to construct AdS analogs of analytic prepotenials of diverse $\mathcal{N}=2$ supergravities on the flat background[3] and to find the higher-spin counterparts of these AdS analytic prepotentials.

3. Effective actions of $6D$ supergauge theories in harmonic superspace

In this Section we review the basic results of our study of the quantum structure of six-dimensional rigid supersymmetric gauge theories. We consider the harmonic superspace formulation of the six-dimensional $\mathcal{N}=(1,0)$ and $\mathcal{N}=(1,1)$ rigid supersymmetric field theories and present the manifestly $\mathcal{N}=(1,0)$ supersymmetric and gauge-invariant methods of constructing off-shell quantum effective action for these theories. It is shown that in $\mathcal{N}=(1,1)$ theory, the one-loop effective action is finite off shell and the off-shell divergences of the two-loop effective action are proportional to the classical equations of motion. Also the results regarding the finite parts of the one-loop low-energy effective action and the divergent structure of the higher-derivative $\mathcal{N}=(1,0)$ supergauge theories are briefly outlined.

3.1. Motivations

Over past few years, our group at BLTP has carried out a series of studies of the quantum structure of supersymmetric gauge theories in six dimensions [43–56]. In this Section of our

[3] The explicit expressions for the linearized conserved higher-spin $\mathcal{N}=2$ supercurrents were presented in a recent paper [42].

survey we briefly describe the methods and approaches developed in our works and the results obtained by making use of these techniques.

The modern interest in the higher-dimensional supersymmetric field theories is inspired by superstring theory. The specific feature of the latter is the existence of the so-called D-branes which amount to the $D+1$ dimensional surfaces in the ten-dimensional space-time. In the low-energy limit the D brane is associated with $D+1$-dimensional supersymmetric gauge theory (see, e.g., [57]). Therefore, the low-energy limit of superstring theory can be described by (extended) supersymmetric quantum field theory in diverse dimensions.

Another motivation to study the higher-dimensional extended supergauge theories is associated with M-theory (see, e.g., [58]). The hypothetical M-theory is characterized by two extended objects: $M2$-brane and $M5$-brane in eleven-dimensional space-time. The field description of interacting multiple $M2$-branes is given by Bagger–Lambert–Gustavsson theory which is $3D$, $\mathcal{N}=8$ supersymmetric gauge theory (see the review [59]). It is expected that the interacting multiple $M5$-branes are related to some new **six**-dimensional extended supersymmetric gauge theory.

The study of the quantum structure of six-dimensional supersymmetric gauge theories dimensionally reduced from superstrings was initiated in works [60, 61]. The one- and two-loop divergences in these theories were studied in [62–68][4]. We wish to pay attention to the fact that practically all considerations in these works have been carried out in the on-shell component approach. The basic goal of our works [43–56] was to develop the approach based on off-shell superfields and to study the off-shell structure of quantum effective action in $6D$ supersymmetric gauge theories.

For description of the six-dimensional supergauge theories, we use the harmonic superfield method. This approach was initially worked out for formulating $4D$ extended supersymmetric theories in terms of unconstrained $\mathcal{N}=2$ superfields [12] (see [15] for details and applications). Harmonic superfield approach was generalized to six dimensions in [70–73]. It allows us to construct the supersymmetric models of gauge multiplet coupled to hypermultiplet in terms of unconstrained $\mathcal{N}=(1,0)$ harmonic superfields [72]. If the hypermultiplet belongs to adjoint representation, such a model possesses an additional hidden on-shell $\mathcal{N}=(0,1)$ supersymmetry and actually describes the maximally extended rigid six-dimensional supersymmetric gauge theory.

The Section is organized as follows. In subsection 3.2, we briefly describe $6D$, $\mathcal{N}=(1,0)$ harmonic superspace and the corresponding $6D$, $\mathcal{N}=(1,0)$ and $6D$, $\mathcal{N}=(1,1)$ supergauge theories in terms of $\mathcal{N}=(1,0)$ harmonic superfields. In subsection 3.3, we discuss the background field method in harmonic superspace which allows one to construct the manifest $\mathcal{N}=(1,0)$ supersymmetric and gauge invariant quantum effective action. In subsection 3.4, we describe calculations of the one-loop divergences in general $\mathcal{N}=(1,0)$ model and show that in $\mathcal{N}=(1,1)$ supersymmetric theory the one-loop divergences vanish off-shell. In subsection 3.5, we describe calculations of the two-loop off-shell divergences and show that they are proportional to classical equations of motion. Subsection 3.6 is devoted to deriving the one-loop low-energy effective action in $\mathcal{N}=(1,1)$ supersymmetric theory. In subsection 3.7, we develop a method to calculate the one-loop divergences in higher-derivative $\mathcal{N}=(1,0)$ supergauge theory. In the concluding subsection 3.8, we summarize the results obtained.

[4]Like in the previous Sections, we basically give here only those references which are directly related to our studies. The full list of references can be found in [43–56].

3.2. $6D$, $\mathcal{N}=(1,1)$ *supersymmetric gauge theory*

Six-dimensional superalgebra is characterized by two independent supercharges forming non-equivalent $6D$ spinors (see, e.g., [69]). The simplest subsets of this algebra are denoted as $\mathcal{N}=(1,0)$ and $\mathcal{N}=(0,1)$ that allows one to formulate the $\mathcal{N}=(1,1)$ supergauge theory as the theory invariant both under $\mathcal{N}=(1,0)$ and under $\mathcal{N}=(0,1)$ supersymmetries. Such a theory is a maximally extended rigid supergauge theory in six dimensions.

The harmonic superspace description of six-dimensional supersymmetry has been given in [70–72]. In this approach, $\mathcal{N}=(1,1)$ theory is formulated as a model of analytic gauge superfield V^{++} coupled to hypermultiplet q^+_A in adjoint representation of gauge group (see the details in [72]). The classical action of such a theory is manifestly $\mathcal{N}=(1,0)$ supersymmetric by construction and has the form

$$\begin{aligned} S_0[V^{++},q^+] \;=\; & \frac{1}{\mathrm{f}_0^2}\Big\{\sum_{n=2}^{\infty}\frac{(-i)^n}{n}\mathrm{tr}\int d^{14}z\,du_1\ldots du_n\frac{V^{++}(z,u_1)\ldots V^{++}(z,u_n)}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)} - \\ & -\frac{1}{2}\mathrm{tr}\int d\zeta^{(-4)}\,q^{+A}\nabla^{++}q^+_A\Big\}, \end{aligned} \tag{3.1}$$

where $\nabla^{++}=D^{++}+iV^{++}$ and f_0 is a dimensional coupling constant ($[\mathrm{f}_0]=m^{-1}$). The analytic superfield V^{++}, describing the gauge multiplet, takes the values in the gauge group algebra,

$$V^{++}=(V^{++})^AT^A, \qquad [T^A,T^B]=if^{ABC}T^C, \qquad A,B,C=1,..,d_G, \tag{3.2}$$

where f^{ABC} are totally antisymmetric structure constants and d_G is the dimension of the gauge group. The generators $T^A_{\mathrm{F}}=t^A$ are normalized in the standard way, $\mathrm{tr}(t^At^B)=\frac{1}{2}\delta^{AB}$.

The action (3.1) is invariant under the following gauge transformations:

$$(V^{++})'=e^{i\lambda^AT^A}V^{++}e^{-i\lambda^AT^A}-ie^{i\lambda^AT^A}D^{++}e^{-i\lambda^AT^A}, \qquad (q^+)'=e^{i\lambda^AT^A}q^+, \tag{3.3}$$

where gauge parameter $\lambda^A(\zeta,u)$ is a real analytic superfield.

Let us introduce the real non-analytic gauge connection $V^{--}=(V^{--})^AT^A$ satisfying the harmonic zero-curvature equation [15]

$$D^{++}V^{--}-D^{--}V^{++}+i[V^{++},V^{--}]=0. \tag{3.4}$$

Using V^{--} one can define the analytic superfield F^{++} [72, 73] as

$$F^{++}\equiv(D^+)^4V^{--}, \qquad \nabla^{++}F^{++}=0. \tag{3.5}$$

The classical equations of motion for the model with action (3.1) read

$$E^{++}=F^{++}+\frac{i}{2}\,[q^{+A},q^+_A]=0, \qquad \nabla^{++}q^+=0. \tag{3.6}$$

The action (3.1) possesses an additional hidden on-shell $\mathcal{N}=(0,1)$ supersymmetry

$$\delta_{(0,1)}V^{++}=\epsilon^{+A}q^+_A, \quad \delta_{(0,1)}q^+_A=-(D^+)^4(\epsilon^-_AV^{--}), \quad \epsilon^{\pm}_A=\epsilon_{aA}\theta^{\pm a}. \tag{3.7}$$

The supersymmetry (3.7) mixes the gauge multiplet and hypermultiplet [72], when hypermultiplet belongs to the same adjoint representation as the gauge multiplet.

3.3. Background field method

Background field method is a procedure to construct quantum effective action in gauge theories in the form preserving classical gauge invariance in quantum theory. For $6D$, $\mathcal{N} = (1, 0)$ supergauge theories such a method was developed in our works [43–45].

In the background field method we split the superfield V^{++} into the sum of the "background" superfields V^{++}, Q^+ and the "quantum" ones v^{++}, q^+,

$$V^{++} \to V^{++} + \mathrm{f}_0 v^{++}; \qquad q^+ \to Q^+ + q^+. \tag{3.8}$$

Then we expand the effective action in a power series in quantum superfields and obtain a theory of the superfields v^{++}, q^+ in the background of the classical superfields V^{++}, Q^+, which are treated as functional arguments of the effective action. Our aim is to study the two-loop contributions to the effective action in the gauge superfield sector. To this end, it is sufficient to assume that the hypermultiplet is purely quantum.

Using the results of refs. [43–45], the general expression for the effective action can be written in the form

$$e^{i\Gamma[V^{++},Q^+]} = \mathrm{Det}^{1/2}\,\widehat{\square} \int \mathcal{D}v^{++}\,\mathcal{D}q^+\,\mathcal{D}b\,\mathcal{D}c\,\mathcal{D}\varphi\,\exp\Big(iS_{\mathrm{total}}\Big), \tag{3.9}$$

where the operator $\widehat{\square} = \frac{1}{2}(D^+)^4(\nabla^{--})^2$, when acting on a space of analytic superfields, is reduced to the covariant superfield d'Alembertian

$$\widehat{\square} = \eta^{MN}\nabla_M\nabla_N + iW^{+a}\nabla^-_a + iF^{++}\nabla^{--} - \frac{i}{2}(\nabla^{--}F^{++}), \tag{3.10}$$

and η_{MN} is $6D$ Minkowski metric with the mostly negative signature. The total action S_{tot} has the form

$$S_{\mathrm{tot}} = \tilde{S} + S_{gf} + S_{FP} + S_{NK}, \tag{3.11}$$

and it includes the gauge-fixing term corresponding to the Feynman gauge,

$$S_{gf}[v^{++}, V^{++}] = -\frac{1}{2}\mathrm{tr}\int d^{14}z du_1 du_2\, \frac{v^{++}_\tau(1)v^{++}_\tau(2)}{(u^+_1 u^+_2)^2} + \tag{3.12}$$

$$+\frac{1}{4}\mathrm{tr}\int d^{14}z du\, v^{++}_\tau(D^{--})^2 v^{++}_\tau, \tag{3.13}$$

the action S_{FP} for the fermionic Faddeev–Popov ghosts b and c, as well as the action S_{NK} for the bosonic real analytic Nielsen–Kallosh ghost φ,

$$S_{FP} = -\mathrm{tr}\int d\zeta^{(-4)}\,\nabla^{++}b\,(\nabla^{++}c + i[v^{++}, c]), \tag{3.14}$$

$$S_{NK} = -\frac{1}{2}\mathrm{tr}\int d\zeta^{(-4)}\,\varphi(\nabla^{++})^2\varphi. \tag{3.15}$$

The label τ of v^{++}_τ means that the superfield is taken in τ-frame [15]. The action (3.13) depends on the background field V^{++} through the background gauge bridge superfield. The action $\tilde{S} = S[V^{++} + \mathrm{f}_0 v^{++}, Q^+ + q^+] - S[V^{++}, Q^+] - \int S'_{V^{++}}\mathrm{f}_0 v^{++} - \int S'_{Q^+}q^+$. Here $S'_{V^{++}}$ and S'_{Q^+} are the functional derivative of the action with respect to V^{++} and Q^+, respectively[5]. This means that the expansion of $\tilde{S}$ with respect of quantum superfields starts with quadratic terms.

[5]The expressions $S'_{V^{++}}\mathrm{f}_0 v^{++}$ and $S'_{Q^+}q^+$ are integrated over harmonic superspace.

3.4. One-loop divergences

The calculation of the effective action is carried out in the framework of the loop expansion. In the one-loop approximation the quantum corrections to the classical action are determined by the quadratic part of the action $S_{\rm tot}$. After integration over quantum superfields this quadratic part produces the one-loop contribution $\Gamma^{(1)}$ to the effective action. In the case under consideration after some transformations one gets the following expression for one-loop contribution to the effective action:

$$\Gamma^{(1)}[V^{++},Q^{+}] = \frac{i}{2}Tr\ln(\widehat{\Box} - 2{\rm f}_0^2 Q^{+}G_{(1,1)}Q^{+}) - \frac{i}{2}Tr\ln(\widehat{\Box}) - \\ - iTr\ln(\nabla^{++})^2_{\rm adj} + \frac{i}{2}Tr\ln(\nabla^{++})^2_{\rm adj} + iTr\ln(\nabla^{++})^2_{R}. \quad (3.16)$$

Here Tr is a functional trace in superspace, $G_{(1,1)}$ is Green function for the operator ∇^{++} (see all details in [44]), and the subscripts adj and R mean the adjoint representation and an arbitrary R representations for the hypermultiplets.

Calculating the one-loop divergences of superfield functional determinants is accomplished in the framework of the proper-time technique (a superfield version of Schwinger – De Witt technique). Such a technique allows us to preserve the manifest gauge invariance and manifest $\mathcal{N}=(1,0)$ supersymmetry at all steps of calculations.

The general scheme of calculations can schematically be summarized as follows:

- Proper-time representation

$$Tr\ln O \sim Tr\int_0^\infty \frac{d(is)}{(is)^{1+\varepsilon}} e^{isO_1}\delta(1,2)|_{2=1}.$$

- Here s is the proper-time parameter and ε is a parameter of dimensional regularization.
- Generically, the total delta-function $\delta(1,2)$ contains the Grassmann delta-function $\delta^8(\theta_1 - \theta_2)$, which vanishes at $\theta_1=\theta_2$.
- Typically, the operator O contains some number of spinor derivatives D^+_a, D^-_a which act on the Grassmann delta-functions $\delta^8(\theta_1-\theta_2)$ and can kill them. A non-zero result is achieved only provided all these δ-functions are killed.
- Only those terms are taken into account which have the pole $\frac{1}{\varepsilon}$ after integration over the proper time.

Result of the calculations can be written in the form

$$\Gamma^{(1)}_{\rm div}[V^{++},Q^{+}] = \frac{C_2 - T(R)}{3(4\pi)^3\epsilon}{\rm tr}\int d\zeta^{(-4)}\,(F^{++})^2 - \\ - \frac{2i{\rm f}_0^2}{(4\pi)^3}\int d\zeta^{(-4)}\,\tilde{Q}^{+}(C_2 - C(R))Q^{+}, \quad (3.17)$$

where $F^{++}=(D^+)^4V^{--}$, ϵ is a parameter of dimensional regularization, and the Casimir operators C_2, $T(R)$, $C(R)$ are defined through the gauge group generators T^A as

$$ {\rm tr}(T^AT^B) = T(R)\delta^{AB},\ (T^A)_m{}^l(T^A)_l{}^n = C(R)\delta_n{}^m,\ C(Adj)_m{}^n = C_2\delta_n{}^m. \quad (3.18)$$

All notations and details of calculations are given in [44]. In the case when the hypermultiplet is in the adjoint representation $T(R)=C(R)=C_2$, the expression (3.17) vanishes. Hence, the $6D$, $\mathcal{N}=(1,1)$ supergauge theory is off-shell finite at one loop.

3.5. Two-loop divergences

Here we discuss a general scheme of singling out the two-loop divergences in $6D$, $\mathcal{N}=(1,1)$ supergauge theory.

- Two-loop divergences are calculated within the background-field method and the proper-time technique like in one-loop case.
- We deal with the vector multiplet background only.
- The power-counting shows that the only possible two-loop divergent contribution in the gauge superfield sector has the following structure [47]:

$$\Gamma^{(2)}_{\rm div}[V^{++}] = a\int d\zeta^{(-4)}\,{\rm tr}\big(F^{++}\,\widehat{\Box}\,F^{++}\big),$$

  with some constant a, which diverges after removing a regularization.
- Within the background field method, the two-loop contributions to the superfield effective action are given by the two-loop vacuum harmonic supergraphs with the background-field dependent lines.
- The background-field dependent propagators (lines) are represented by the proper-time integrals.

A direct calculation [47] yields the following result for the off-shell divergent part of the two-loop contribution $\Gamma^{(2)}[V^{++}]$ to effective action in the gauge-superfield sector:

$$\Gamma^{(2)}_{\rm div} = \frac{8{\rm f}_0^2}{(4\pi)^6\varepsilon^2}(C_2)^2{\rm tr}\int d\zeta^{(-4)}\,F^{++}\,\widehat{\Box}\,F^{++}. \tag{3.19}$$

Here we have presented only those two-loop divergences which contain the two-loop pole $\frac{1}{\varepsilon^2}$. Calculating the sub-leading divergences corresponding to the simple pole $\frac{1}{\varepsilon}$ remains an open issue at present.

The hypermultiplet-dependent contribution to the two-loop divergences can be found by the straightforward quantum computations of the two-loop effective action, taking into account the hypermultiplet background Q^+. However, the general form of the hypermultiplet-dependent divergences can in principle be found without direct calculations, just assuming the invariance of the effective action under the hidden $\mathcal{N}=(0,1)$ supersymmetry. The result has the extremely simple form

$$\Gamma^{(2)}_{\rm div}[V^{++}, Q^+] = a\int d\zeta^{(-4)}\,{\rm tr}E^{++}\,\widehat{\Box}\,E^{++}, \tag{3.20}$$

where $E^{++} = F^{++} + \frac{i}{2}[Q^{+A}, Q^+_A]$ is the classical equation of motion for the gauge superfield coupled to the hypermultiplet. It is evident that the two-loop divergences vanish on-shell. Let us pay attention to the fact that the two-loop divergences can be canceled by means of the off-shell redefinition $V^{++} \to V^{++} + a\,\widehat{\Box}\,E^{++}$ in the classical action.

3.6. Low-energy effective action in $6D$, $\mathcal{N}=(1,1)$ *theory*

The structure of finite low-energy contributions to the one-loop effective action $\Gamma^{(1)}[V^{++}]$ was discussed in [46, 51]. It was shown that the simplest way to carry out the calculations of the low-energy effective action is to use the formulation of $6D$, $\mathcal{N}=(1,1)$ supergauge theory in terms of ω-hypermultiplet [15]. In this case, the q-hypemultiplet-dependent term in the classical action (3.1) is replaced by the ω-hypemultiplet term of the form

$$-\frac{1}{2\mathrm{f}^2}\mathrm{tr}\int d\zeta^{(-4)}\,\nabla^{++}\Omega\nabla^{++}\Omega, \tag{3.21}$$

where f is a coupling constant. The first V^{++}-dependent term in the action (3.1) does not change its form. The superfield Ω takes values in the adjoint representation of gauge group $SU(N)$ and $\nabla^{++}\Omega=D^{++}\Omega+i[V^{++},\Omega]$.

The effective action $\Gamma[V^{++}]$ is constructed within the background field method, where the initial superfields V^{++} and Ω are split into the background superfields $\mathbf{V}^{++}$, $\mathbf{\Omega}$, and the quantum superfields v^{++}, ω by the substitution $V^{++}\to\mathbf{V}^{++}+\mathrm{f}v^{++}$, $\Omega\to\mathbf{\Omega}+\mathrm{f}\omega$. Details of the background method in this case are given in [46, 51].

For further consideration we assume that the background superfields align in a fixed direction inside the Cartan subalgebra of $su(N)$

$$\mathbf{V}^{++}=V_0^{++}H,\quad \mathbf{\Omega}=\Omega_0H, \tag{3.22}$$

where H is a fixed generator of the Cartan subalgebra which generates some Abelian subgroup $U(1)$. The choice (3.22) corresponds to the spontaneous symmetry breaking $SU(N)\to SU(N-1)\times U(1)$. Note that the two background superfields (V_0^{++},Ω_0) form the Abelian vector $\mathcal{N}=(1,1)$ multiplet. The bosonic sector of such a multiplet contains a single real gauge vector field A_M and four real scalars ϕ and $\phi^{(ij)}$ with $i,j=1,2$. The fields A_M, ϕ, $\phi^{(ij)}$ in six dimensions describe the bosonic world-volume degrees of freedom of the $D5$-brane [75, 76].

The effective action is calculated for the Abelian background superfields V_0^{++}, Ω_0 satisfying the classical equations of motion $F_0^{++}=0$, $(D^{++})^2\Omega_0=0$ and slowly varying in space-time, $\partial_MW_0^{+}=0$, $\partial_M\Omega_0=0$. Applying the background method and superfield proper-time technique leads to the leading low-energy correction to the effective action:

$$\Gamma^{(1)}_{\mathrm{lead}}=\frac{N-1}{(4\pi)^3}\int d\zeta^{(-4)}\,\frac{(W_0^{+})^4}{\Omega_0^2}. \tag{3.23}$$

In the bosonic sector the effective action (3.23) yields

$$\Gamma^{(1)}_{\mathrm{bosonic}}\sim\int d^6x\frac{F^4}{\phi^2}(1+\frac{\phi^{(ij)}\phi_{(ij)}}{\phi^2}), \tag{3.24}$$

where $F^4=F_{MN}F^{MN}F_{PQ}F^{PQ}-4F^{NM}F_{MR}F^{RS}F_{SN}$ and F_{MN} is the Abelian gauge strength.

3.7. One-loop divergences in higher-derivative $6D$, $\mathcal{N}=(1,0)$ *supergauge theory*

The higher-derivative $6D$, $\mathcal{N}=(1,0)$ supersymmetric gauge theory was formulated in harmonic superspace in [73]. The classical action of this theory reads

$$S[V^{++}]=\pm\frac{1}{2g_0^2}\mathrm{tr}\int d\zeta^{(-4)}\,(F^{++})^2, \tag{3.25}$$

where F^{++} was defined in (3.5) and g_0 is a dimensionless coupling constant[6]. At the component level such a model contains four space-time derivatives in the bosonic sector. In particular, the component action includes the term $\sim \int d^6x \mathrm{tr}(\nabla^M F_{MN})^2$, with F_{MN} being the standard Yang–Mills strengths. Aspects of the renormaization of this theory were studied in [73, 74], and the one-loop counterterms were calculated there in the component approach. The full-fledged superfield description of the one-loop divergences was given in [52, 53].

Harmonic superfield quantization of the theory under consideration is carried out in the framework of the background field method developed for this case in [52, 53]. The method assumes the background-quantum splitting $V^{++} \to V^{++} + v^{++}$, where V^{++}, v^{++} in the right-hand side are background and quantum superfields, respectively, with imposing the gauge-fixing conditions on v^{++}. These conditions are taken to be background-field dependent, in such a way that the effective action remains gauge invariant. In the one-loop case the effective action is calculated using the quadratic parts of the classical action and ghost actions. All details can be found in [52, 53].

Here we present only the final result for the divergences of the one-loop effective action

$$\Gamma^{(1)}_{\mathrm{div}} = -\frac{4C_2}{\epsilon(4\pi)^3} \int d\zeta^{(-4)}\, \mathrm{tr}(F^{++})^2, \tag{3.26}$$

with ϵ being a parameter of dimensional regularization, and C_2 is the second Casimir operator of $SU(N)$ group. The superfield result (3.26) matches the component calculations of refs. [73, 74][7].

Comparing the classical action (3.25) with the one-loop divergent quantum correction, we can write the one-loop renormalization relation between the bare coupling constant g_0 and the renormalized one g in the form

$$\frac{1}{g^2} = \frac{1}{g_0^2} \mp \frac{22C_2}{3\epsilon(4\pi)^3}. \tag{3.27}$$

The relations (3.25) and (3.27) imply that the one-loop β-function in the theory under consideration has the form

$$\beta(\alpha) = \mp \frac{11\alpha^2 C_2}{24\pi^2}, \tag{3.28}$$

where $\alpha = \frac{g^2}{4\pi}$. The lower sign in relation (3.28) corresponds to the Landau pole, while the upper sign corresponds to asymptotic freedom.

3.8. Summary and outlook

Let us summarize. In this Section, we presented a brief review of works devoted to studying the quantum structure of six-dimensional supersymmetric gauge theories. Our study was focused on the development of manifestly supersymmetric and gauge invariant methods of construction of the effective action in $6D$, $\mathcal{N} = (1,0)$ and $\mathcal{N} = (1,1)$ super Yang–Mills theories and in $6D$, $\mathcal{N} = (1,0)$ higher-derivative supergauge theory.

[6] In the conventional field theory the overall sign of the action is fixed by the requirement of positiveness of energy. However, in higher-derivative theories the energy is not positively defined in general. Therefore, in the case under consideration there are no reasons to fix the overall sign of the action.

[7] The result (3.26) contains contributions from both gauge multiplet and ghost loops (see [52, 53]).

The theories under consideration are formulated in six-dimensional harmonic superspace which provides the manifest $\mathcal{N} = (1,0)$ supersymmetry. The quantum effective action is defined in the framework of the background superfield method allowing one to carry out the calculations with preserving both $\mathcal{N} = (1,0)$ supersymmetry and gauge invariance of the effective action. Besides, we developed a power counting in the harmonic superspace which ensures, along with the manifest $\mathcal{N} = (1,0)$ supersymmetry and explicit gauge invariance, the explicit form of possible counterterms up to numerical coefficients.

As a result, we have found the off-shell one-loop divergences and shown that there exists a gauge in which all the one-loop divergences of $\mathcal{N} = (1,1)$ theory completely vanish off shell. As the next step, we have studied the two-loop divergences in this theory. Due to the one-loop finiteness, the two-loop supergraphs do not contain the divergent subgraphs, which essentially simplifies the analysis of the two-loop counterterms. It has been shown that at both one-loop and two-loop levels the off-shell counterterms are built only from the classical equations of motion and, hence, vanish on shell.

Using the harmonic superspace background field method we have developed an approach to construct the low-energy effective action in $\mathcal{N} = (1,1)$ supergauge theory. Such an effective action is defined for the slowly varying background fields corresponding to spontaneous symmetry breaking of the gauge group $SU(N) \to SU(N-1) \times U(1)$. The calculations were carried out in the framework of the superfield proper-time technique.

The methods developed were applied to studying the divergence structure of the one-loop effective action in a higher-derivative $\mathcal{N} = (1,0)$ super Yang–Mills theory. In the framework of the off-shell supersymmetric and gauge invariant approach the one-loop renormalization of the coupling constant was derived and the corresponding β-function was found.

The results of the series of our works [43–56] clearly demonstrate the power of the harmonic superspace approach for formulation of six-dimensional supersymmetric gauge theories and for the study of the structure of quantum effective actions of these theories. These findings suggest a few opportunities to explore a number of open questions related to the structure of effective action in $6D$, $\mathcal{N} = (1,1)$ supergauge theories. Firstly, it is important to determine the structure of three-loop divergences, which, unlike one- and two-loop ones, should not disappear on the mass shell. Secondly, it would be extremely interesting to study the possibility of an additional hidden $\mathcal{N} = (0,1)$ supersymmetry in the gauge multiplet-hypermultiplet higher-derivative system.

Acknowledgements

The authors are grateful to Academician V. A. Matveev for the suggestion to prepare this review and to S. A. Fedoruk for useful remarks. We thank B. S. Merzlikin, K. V. Stepanyantz, and N. M. Zaigraev for a close collaboration on the topics overviewed here.

Conflicts of interest

The authors declare no conflicts of interest.

References

[1] E. A. Ivanov, Gauge fields, nonlinear realizations, superymmetry, Phys. Part. Nucl. 47 (2016) 4, 508–539; arXiv:1604.01379[hep-th].

[2] V. I. Ogievetsky, I. V. Polubarinov, The notoph and its possible interactions, Yad. Fiz. 4 (1966) 216–223 [Sov. J. Nucl. Phys. 4 (1967) 156–161].

[3] V. I. Ogievetsky, Infinite-dimensional algebra of general covariance group as the closure of finite-dimensional algebras of conformal and linear groups, Lett. Nuovo Cim. 8 (1973) 988–990.

[4] A. B. Borisov, V. I. Ogievetsky, Theory of dynamical affine and conformal symmetries as gravity theory, Teor. Mat. Fiz. 21 (1974) 329–342 [Theor. Math. Phys. 21 (1975) 1179–1188].

[5] E. A. Ivanov, V. I. Ogievetsky, The inverse Higgs phenomenon in nonlinear realizations, Teor. Mat. Fiz. 25 (1975) 164–177.

[6] V. Ogievetsky, E. Sokatchev, Structure of supergravity group, Phys. Lett. B 79 (1978) 222–224.

[7] V. Ogievetsky, E. Sokatchev, The gravitational axial superfield and the formalism of differential geometry, Yad. Fiz. 31 (1980) 821–840 [Sov. J. Nucl. Phys. 31 (1980) 424].

[8] E. A. Ivanov, A. A. Kapustnikov, General relationship between linear and nonlinear realizations of supersymmetry, J. Phys. A 11 (1978) 2375–2384.

[9] E. A. Ivanov, A. A. Kapustnikov, The nonlinear realization structure of models with spontaneously broken supersymmetry, J. Phys. G 8 (1982) 167–191.

[10] A. Galperin, E. Ivanov, V. Ogievetsky, Grassmann analyticity and extended supersymmetry, Pis'ma ZhETF 33 (1981) 176–181 [JETP Lett. 33 (1981) 168–172].

[11] A. Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic superspace: Key to $N = 2$ supersymmetric theories, Pis'ma ZhETF 40 (1984) 155–158 [JETP Lett. 40 (1984) 912–916].

[12] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky, E. Sokatchev, Unconstrained $N = 2$ matter, Yang–Mills and supergravity theories in harmonic superspace, Class. Quant. Grav. 1 (1984) 469–498; Class. Quant. Grav. 2 (1985) 127 E.

[13] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky, E. Sokatchev, Unconstrained off-shell $N = 3$ supersymmetric Yang–Mills theory, Class. Quant. Grav. 2 (1985) 155–166.

[14] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky, E. Sokatchev, $N = 3$ supersymmetric gauge theory, Phys. Lett. B 151 (1985) 215–218.

[15] A. S. Galperin, E. A. Ivanov, V. I. Ogievetsky, E. S. Sokatchev, Harmonic superspace, Cambridge University Press 2001, 306 p.

[16] C. Fronsdal, Massless fields with integer spin, Phys. Rev. D 18 (1978) 3624.

[17] J. Fang, C. Fronsdal, Massless fields with half integral spin, Phys. Rev. D 18 (1978) 3630.

[18] T. Courtright, Massless field supermultiplets with arbitrary spins, Phys. Lett. B 85 (1979) 2019.

[19] M. A. Vasiliev, Gauge form of description of massless fields with arbitrary spin, Yad. Fiz. 32 (1980) 855–861 (in Russian), Sov. J. Nucl. Phys. 32 (1980) 439.

[20] S. Kuzenko, A. Sibiryakov, V. Postnikov, Massless gauge superfields of higher half integer superspins, JETP Lett. 57 (1993) 534; S. Kuzenko, A. Sibiryakov, Massless gauge superfields of higher integer superspins, JETP Lett. 57 (1993) 539.

[21] S. M. Kuzenko, A. G. Sibiryakov, Free massless higher superspin superfields on the anti-de Sitter superspace, Phys. Atom. Nucl. 57 (1994) 1257–1267; arXiv:1112.4612[hep-th].

[22] I. Buchbinder, E. Ivanov and N. Zaigraev, Unconstrained off-shell superfield formulation of $4D$, $\mathcal{N} = 2$ supersymmetric higher spins, JHEP 12 (2021) 016, arXiv:2109.07639[hep-th].

[23] I. Buchbinder, E. Ivanov and N. Zaigraev, Off-shell cubic hypermultiplet couplings to $\mathcal{N} = 2$ higher spin gauge superfields, JHEP 05 (2022) 104, arXiv:2202.08196[hep-th].

[24] I. Buchbinder, E. Ivanov and N. Zaigraev, $\mathcal{N} = 2$ higher spins: Superfield equations of motion, the hypermultiplet supercurrents, and the component structure, JHEP 03 (2023) 036, arXiv:2212.14114[hep-th].

[25] I. Buchbinder, E. Ivanov and N. Zaigraev, $\mathcal{N} = 2$ superconformal higher-spin multiplets and their hypermultiplet couplings, JHEP 08 (2024) 120, arXiv:2404.19016[hep-th].

[26] E. Ivanov and N. Zaigraev, $\mathcal{N} = 2$ AdS hypermultiplets in harmonic superspace, Phys. Lett. B 871 (2025) 139964, arXiv:2509.01406 [hep-th].

[27] E. Ivanov and N. Zaigraev, $\mathcal{N} = 2$ superconformal gravitino in harmonic superspace, Phys. Lett.

B 862 (2025) 139333, arXiv:2412.14822[hep-th].

[28] R. Manvelyan, K. Mkrtchyan, W. Ruehl, A generating function for the cubic interactions of higher spin fields, Phys. Lett. B 696 (2011) 410–415, arXiv:1009.1054[hep-th].

[29] R. R. Metsaev, Cubic interaction vertex for $\mathcal{N}=1$ arbitrary spin massless supermultiplet in flat space, JHEP 08 (2019) 130, arXiv:1905.11357[hep-th].

[30] R. R. Metsaev, Cubic interactions for arbitrary spin $\mathcal{N}$-extended massless supermultiplets in $4d$ flat space, JHEP 11 (2019) 084, arXiv:1909.05241[hep-th].

[31] E. S. Fradkin and A. A. Tseytlin, Conformal supergravity, Phys. Rep. 119 (1985) 233–362.

[32] E. S. Fradkin, V. Y. Linetsky, Cubic interaction in conformal theory of integer higher spin fields in four-dimensional space-time, Phys. Lett. B 231 (1989) 97–106.

[33] E. S. Fradkin, V. Y. Linetsky, Superconformal higher spin theory in the cubic approximation, Nucl. Phys. B 350 (1991) 274–324.

[34] A. Y. Segal, Conformal higher spin theory, Nucl. Phys. B 664 (2003) 59–130, arXiv:hep-th/0207212[hep-th].

[35] S. M. Kuzenko, R. Manvelyan, S. Theisen, Off-shell superconformal higher spin multiplets in four dimensions, JHEP 07 (2017) 034, arXiv:1701.00682 [hep-th].

[36] S. M. Kuzenko and E. S. N. Raptakis, Extended superconformal higher-spin gauge theories in four dimensions, JHEP 12 (2021) 210, arXiv:2104.10416[hep-th].

[37] E. A. Ivanov, A. S. Sorin, Superfield formulation of $OSp(1,4)$ supersymmetry, J. Phys. A 13 (1980) 1159–1188.

[38] S. M. Kuzenko, G. Tartaglino-Mazzucchelli, Five-dimensional $\mathcal{N}=1$ AdS superspace: Geometry, off-shell multiplets and dynamics, Nucl. Phys. B 785 (2007) 34–73, arXiv:0704.1185[hep-th].

[39] E. I. Buchbinder, S. M. Kuzenko, and I. B. Samsonov, Massless higher-spin supermultiplets in $5D$ harmonic superspace, JHEP 02 (2026) 122, arXiv:2509.06604[hep-th].

[40] S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, Superspace or one thousand and one lessons in supersymmetry, Front. Phys. 58 (1983) 1–548, arXiv:hep-th/0108200[hep-th].

[41] I. Bandos, E. Ivanov, J. Lukierski, D. Sorokin, On the superconformal flatness of AdS superspaces, JHEP 06 (2002) 040, arXiv:hep-th/0205104[hep-th].

[42] N. Zaigraev, $\mathcal{N}=2$ higher-spin supercurrents, Phys. Lett. B 858 (2024) 139056, arXiv:2408.00668[hep-th].

[43] I. L. Buchbinder, E. A. Ivanov, M. B. Merzlikin, K. V. Stepanyantz, One-loop divergences in the $6D$, $\mathcal{N}=(1,0)$ Abelian gauge theory, Phys. Lett. B 763 (2016) 375, arXiv:1609.00975[hep-th].

[44] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, One-loop divergences in $6D$, $N=(1,0)$ SYM theory, JHEP 1701 (2017) 128, arXiv:1612.03190[hep-th].

[45] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, Supergraph analysis of the one-loop divergences in $6D$, $\mathcal{N}=(1,0)$ and $\mathcal{N}=(1,1)$ gauge theories, Nucl. Phys. B 921 (2017) 127, arXiv:1704.02530[hep-th].

[46] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, Leading low-energy effective action in $6D,\mathcal{N}=(1,1)$ and $\mathcal{N}=(1,1)$ SYM theory, JHEP 09 (2018) 039, arXiv:1711.03302[hep-th].

[47] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, On the two-loop divergences of the 2-point hypermultiplet supergraphs for $6D$, $\mathcal{N}=(1,1)$ and $\mathcal{N}=(1,1)$ SYM theory, Phys. Lett. B 778 (2018) 252, arXiv:1711.11514[hep-th].

[48] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, Gauge dependence of the one-loop divergences in $6D,\mathcal{N}=(1,0)$ Abelian theory, Nucl. Phys. B 936 (2018) 638, arXiv:1808.08446[hep-th].

[49] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, Harmonic superspace approach to the effective action in six-dimensional supersymmetric gauge theories, Symmetry 11 (2019) 1, arXiv:1812.02681[hep-th].

[50] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, On gauge dependence of the one-loop divergences in $6D$ $\mathcal{N} = (1,0)$ and $\mathcal{N} = (1,1)$ SYM theories, Phys. Lett. B 798 (2019) 134957, arXiv:1907.12302[hep-th].

[51] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, Low-energy $6D$, $\mathcal{N} = (1,1)$ SYM effective action beyond the leading approximation, Nucl. Phys. B 954 (2020) 114995, arXiv:1912.02634[hep-th].

[52] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, Supergraph calculation of one-loop divergences in higher-derivative $6D$ SYM theory, JHEP 08 (2020) 169, arXiv:2004.12657[hep-th].

[53] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, The renormalization structure of $6D$, $\mathcal{N} = (1,0)$ supersymmetric higher-derivative gauge theory, Nucl. Phys. B 961 (2020) 115249, arXiv:2007.02843[hep-th].

[54] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, On the two-loop divergences in $6D$, $\mathcal{N} = (1,1)$ SYM theory, Phys. Lett. B 820 (2021) 136516, arXiv:2104.14284[hep-th].

[55] A. S. Budekhina, I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, On two-loop divergences in $6D$, $\mathcal{N} = (1,1)$ supergauge theory, Phys. Part. Nucl. Lett. 19 (2022) 666.

[56] I. L. Buchbinder, E. A. Ivanov, B. S. Merzlikin, K. V. Stepanyantz, On two-loop divergences of effective action in $6D$, $\mathcal{N} = (1,1)$ SYM theory, JHEP 05 (2023) 089, arXiv:2212.03766[hep-th].

[57] C. V. Johnson, D-branes, Cambridge Univ. Press, 2006.

[58] K. Becker, M. Becker, J. H. Schwarz, String theory and M-theory, Cambridge Univ. Press, 2012.

[59] J. Bagger, N. Lambert, S. Mukhi, C. Papageorgakis, Multiple membranes in M-theory, Phys. Rep. 527 (2013) 1, arXiv:1203.3546[hep-th].

[60] N. Seiberg, E. Witten, Comments on string dynamics in six dimensions, Nucl. Phys. B 471 (1996) 121, arXiv:hep-th/9603003[hep-th].

[61] N. Seiberg, New theories in six dimensions and matrix description of M-theory on T^5 and T^5/Z_2, Phys. Lett. B 408 (1997) 98, arXiv:hep-th/9705221[hep-th].

[62] E. S. Fradkin, A. A. Tseytlin, Quantum properties of higher dimensional and dimensionally reduced supersymmetric theories, Nucl. Phys. B 227 (1983) 252.

[63] P. S. Howe, K. S. Stelle, Ultraviolet divergences in higher dimensional supersymmetric Yang–Mills theories, Phys. Lett. B 137 (1984) 175.

[64] N. Marcus, A. Sagnotti, A test of finiteness predictions for supersymmetric theories, Phys. Lett. B 135 (1984) 85.

[65] N. Marcus, A. Sagnotti, The ultraviolet behavior of $N = 4$ Yang–Mills and the power counting of extended super-space, Nucl. Phys. B 256 (1985) 77.

[66] P. S. Howe, K. S. Stelle, Supersymmetry counterterms revisited, Phys. Lett. B 554 (2003) 190, arXiv:hep-th/0211279[hep-th].

[67] D. I. Kazakov, Ultraviolet fixed points in gauge and SUSY field theories in extra dimensions, JHEP 03 (2003) 020, arXiv:hep-th/0209100[hep-th].

[68] L. V. Bork, D. I. Kazakov, M. V. Kompaniets, D. M. Tolkachev, D. E. Vlasenko, Divergences in maximal supersymmetric Yang–Mills theories in diverse dimensions, JHEP 11 (2015) 059, arXiv:1508.05570[hep-th].

[69] P. S. Howe, G. Sierra, P. K. Townsend, Supersymmetry in six dimensions, Nucl. Phys. B 221 (1983) 331.

[70] P. S. Howe, K. S. Stelle, P. C. West, $N = 1$ $d = 6$ harmonic superspace, Class. Quant. Grav. 2 (1985) 815.

[71] B. M. Zupnik, Six-dimensional supergauge theories in the harmonic superspace, Sov. J. Nucl. Phys. 44 (1986) 512 [Yad. Fiz. 44 (1986) 794].

[72] G. Bossard, E. Ivanov and A. Smilga, Ultraviolet behavior of $6D$ supersymmetric Yang–Mills

theories and harmonic superspace, JHEP 12 (2015) 085, arXiv:1509.08027[hep-th].

[73] E. A. Ivanov, A. V. Smilga, B. M. Zupnik, Renormalizable supersymmetric gauge theory in six dimensions, Nucl. Phys. B 726 (2005) 131, arXiv:0505082[hep-th].

[74] A. Casarin, A. A. Tseytlin, One-loop β-function in 4-derivative gauge theory in 6 dimensions, JHEP 08 (2019) 159, arXiv:1907.02501[hep-th].

[75] A. Giveon, D. Kutasov, Brane dynamics and gauge theory, Rev. Mod. Phys. 71 (1999) 981, arXiv:hep-th/9802067[hep-th].

[76] R. Blumenhagen, B. Kors, D. Lüst, S. Stieberger, Four-dimensional string compactifications with D-branes, orbifolds and fluxes, Phys. Rep. 445 (2007) 1–193, arXiv:hep-th/0610327[hep-th].