

High Energy Nuclear Optics of polarized nucleons and nuclei: research at the Nuclotron-M/NICA complex

V. G. Baryshevsky *¹

¹*Institute for Nuclear Problems, Belarusian State University, Minsk, Belarus*

Abstract

Refraction of particles (nucleons, nuclei, γ -quanta) in matter with polarized protons (nuclei) results in revealing coherent quasi-optical phenomenon of nuclear spin precession of particles (nuclei) in the pseudomagnetic field of matter with polarized spins and the phenomenon of birefringence of particles (nuclei) with spin $S \geq 1$. These phenomena can be observed and studied at the Nuclotron-M/NICA complex. The similar effects for γ -quanta could be observed at the LINAC accelerator. Quasi-optical coherent phenomena of spin rotation and dichroism are not caused by strong interactions only, the T-odd P-odd, T-odd P-even, and T-even P-odd interactions also contribute. Limits on the values of these contributions at the energies available at the Nuclotron-M/NICA complex can be obtained by investigating all these phenomena. When studying polarized particle collisions, it is necessary to consider possible influences of quasi-optical phenomena of spin rotation and spin dichroism caused by nuclear precession and birefringence.

Keywords: coherent quasi-optical phenomena, spin rotation, spin dichroism, birefringence

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Contents

1. Introduction: brief history	2
2. Light optics	5
3. Gamma optics	7
3.1. Quasi-optical phenomenon of γ -quanta polarization plane rotation in matter with polarized proton (nuclei)	9
3.2. Quasi-optical birefringence effect for γ -quanta in matter with polarized nuclei	15
4. Quasi-optical phenomena for low-energy particles	17
4.1. Neutron Spin Precession in a Pseudomagnetic Field of Matter with Polarized Nuclei	17
4.2. Growth of Nuclear Spin Precession Frequency of Antiprotons (Negative Hyperons) Under Deceleration in Matter with Polarized Nuclei	23
4.3. Forward Scattering Amplitude of Negatively Charged Particles	24
4.4. Effective Potential Energy of Negatively Charged Particles in Matter	26

*Corresponding author e-mail address: v_baryshevsky@yahoo.com

4.5. Spin Rotation Angle of Low-Energy Antiprotons in Polarized Matter	28
4.6. Mirror Reflection of Antiprotons from the Surface of Matter	30
4.7. The Amplitude of Antiproton Scattering by a Nucleus (Atom) at Low Energy	32
4.8. Spin Polarization of Antiprotons Reflected from the Vacuum-Matter Boundary	34
5. Quasi-optical Phenomena for High-energy Particles in Polarized Matter	37
5.1. The effect of spin precession at high energies	39
5.2. Spin Rotation and Spin Dichroism	40
5.3. Proton (Antiproton) Spin Rotation in a Thick Polarized Target and Spin Filtering of Particle Beams in a Nuclear Pseudomagnetic Field	42
5.4. Spin density matrix formalism	45
6. The phenomenon of birefringence (spin oscillation and spin dichroism) of particles with spin $S \geq 1$	50
6.1. Rotation and Oscillation of Deuteron Spin in Nonpolarized Matter and Spin Dichroism (Birefringence Phenomenon)	54
6.2. The Effect of Tensor Polarization Emerging in Nonpolarized Beams Moving in Nonpolarized Matter	58
6.3. The Amplitude of Zero-Angle Elastic Scattering of a Deuteron by a Nucleus	60
6.4. First Observation of Spin Dichroism with Deuterons in a Carbon Target	63
6.5. Tensor polarization rotation and oscillation between vector and tensor polarizations	64
6.6. Dichroism effect, rotation of spin and tensor polarization for heavy nuclei	66
6.7. Possibilities to observe spin dichroism at the internal target of the Nuclotron-M	67
6.8. Evolution of polarization characteristics of a particle beam in an internal target of Nuclotron	71
6.9. Dichroism effect for a deuteron beam moving in the Nuclotron with internal target	73
6.10. About Possible Influence of Birefringence Effect on the Processes of Production (Photoproduction, Electroproduction) of Vector Mesons in Nuclei	75
7. "Optical" spin rotation and birefringence of particles at NICA	76
7.1. Mutual refraction of colliding beams	77
7.2. Evaluations of spin dichroism effect at beam collisions at NICA	79
8. Conclusion	82

1. Introduction: brief history

In the autumn of 1963, in November, for the first time I came to JINR for a few days. There, in the Laboratory of Theoretical Physics, I, then a graduate student of the Department of Nuclear Physics at the Physics Faculty of the Belarusian University named after V. I. Lenin, was to have an interview with Semyon Solomonovich Gershtein. After the interview, I was introduced to Yakov Abramovich Smorodinsky, who told me to come in January 1964 and start working. In the first days of January, following Semyon Solomonovich assignment, I began to acquaint myself with topics related to non-leptonic decays of hyperons. However, about a week later, an event occurred in my life that turned me into a physicist and bound me to JINR for my entire life. In mid-January, Semyon Solomonovich told me that there was an interesting problem related to measuring the mass difference of K_L and K_S mesons, and that more details would be explained to me at the Laboratory of High Energy Physics (LHEP). We went there, entered Veksler's office, and Semyon Solomonovich introduced me to the person who turned me into a physicist. That person was Mikhail Isaakovich Podgoretsky. I was amazed because in the autumn of 1963 an article by Podgoretsky and Khrustalev, devoted to the discussion of

interference phenomena in quantum mechanics, was published in UFN (Soviet Physics Uspekhi). After reading it, I said to myself — this is what I would like to do. The hand of fate led me to Mikhail Isaakovich. Our work started very quickly. After obtaining the first results, we were to present them at a seminar at LHEP. But before the presentation, Yakov Borisovich Zeldovich phoned and said that everything we were planning to talk about had already been done by him. Mikhail Isaakovich was upset, yet his remarkable intuition forced him to say to me: “Volodya, it can’t be that the system of K_L and K_S mesons is unique... Go to the library and dig into the literature”.

After some investigations, I came across neutron interaction with polarized nuclei. We then began studying the interaction of slow neutrons with a polarized nuclear target. After conducting some theoretical calculations, I found that the neutron spin in such a target would undergo rotation. With this result, I went to Mikhail Isaakovich. During our discussion, he said an important phrase that immediately evinced the nature of the phenomenon. He explained that this behavior of the neutron indicates the presence of an effective pseudomagnetic field caused by the nuclear interactions of the neutron with the ensemble of polarized nuclei in the target, which affects the neutron. This pseudomagnetic nuclear field should be added to the conventional magnetic field of the target, which is produced by the magnetic moments of the nuclei. For thermal neutrons in a target with polarized protons, the nuclear pseudomagnetic field is two orders of magnitude greater than the ordinary magnetic field produced by the proton magnetic moments. After that, in just a couple of weeks, we wrote an article and submitted it to the Journal of Experimental and Theoretical Physics (JETP), where it was published in the same year 1964. Later, the effect of nuclear spin precession of neutrons in a target with polarized nuclei was registered as USSR discovery No. 224.

The next significant result was obtained jointly by me and Vladimir Lvovich Lyuboshits. In an article published in JETP in 1965, we showed that for γ -quanta passing through matter with polarized electrons, a quasi-optical effect of polarization plane rotation occurs, which is kinematically analogous to the Faraday effect for light. This result was later registered as USSR discovery No. 360.

In my next work, published in JETP in 1966, it was shown that for γ -quanta passing through a target with polarized nuclei, a birefringence effect similar to the birefringence effect of light also exists.

How was it possible that a young man of 23 years old, having arrived in Dubna, completed several highly interesting works in just a few years? The answer is quite simple: at that time, JINR had an amazing creative atmosphere. I took a place in Veksler’s office. This office was given to M. I. Podgoretsky. Many JINR employees from different laboratories came to our office. Every week Nikolay Nikolayevich Bogolyubov held seminars at the Laboratory of Theoretical Physics. There were no problems in consulting any laboratory.

The most astonishing event for me happened during lunch at the restaurant in the hotel “Dubna”.

Once, during lunch, an elderly man took a seat at my table. He asked me questions and showed genuine interest in my results. The next day, the stranger came to my table again. We had lunch together and continued our conversations. Finally, after a few days, he introduced himself — he was Georgy Flerov — and invited me to visit him at the Laboratory of Nuclear Reactions (LNR). This was an incredible event in my ordinary life.

Discussions and talks went on continuously. Certainly, in such an extremely intellectual incubator, many interesting thoughts and ideas naturally arose. Soon I established scientific contacts that grew into friendship with I. M. Frank. I was a reviewer of his book dedicated to

the Vavilov-Cherenkov effect, and we had a joint article published in the Journal of Nuclear Physics. Bruno Maksimovich Pontecorvo became the opponent of my doctoral dissertation in 1973.

And, of course, during all this time I continued active joint scientific work with my teacher, M. I. Podgoretsky. We tried to attract the interest of neutron physicists in performing experiments on nuclear precession. However, Fyodor Lvovich Shapiro at that time was fully occupied with preparing experiments aimed at obtaining ultracold neutrons.

The arrival of Prof. A. Abraham in Dubna dramatically changed the situation. A. Abraham was greatly interested in the phenomenon, and after returning to France he immediately conducted an experiment. Naturally, the effect was experimentally discovered. At the same time, in Italy, the nuclear precession effect was discovered by Prof. M. Forte.

The effect of polarization plane rotation for γ -quanta in a target with polarized electrons was observed by V. M. Lobashev. The effect manifested itself as a kind of noise in Lobashev's experiments. He discussed the problem with Semyon Solomonovich Gershtein, who promptly understood that Lobashev had discovered our phenomenon. S. S. Gershtein summoned me and sat me down to analyze Lobashev's experiment in order to determine whether he had truly observed the effect of polarization plane rotation for γ -quanta. The analysis showed that this was indeed the case. Thus, Semyon Solomonovich, with his inherent clear understanding of physics, helped to perceive manifestations of the rotation effect in Lobashev's experimental results. As a result, the rotation effect was really experimentally detected.

The birefringence effect for γ -quanta has not yet been detected. However, at present, due to the development of techniques for creating polarized nuclear targets, the conditions for its detection have matured. This could, in particular, be done in Dubna with the LINAC-800 accelerator within the framework of the FLAP collaboration.

As noted, the effect of nuclear spin precession for neutrons is kinematically analogous to the Faraday effect. An interesting question arises: Is there an effect for particles (nuclei) similar to the birefringence effect for light (γ -quanta)?

It turns out that such an effect exists and arises when a particle with spin $S \geq 1$ moves through a nonpolarized! target. This effect arises due to the internal anisotropy of particles with spin $S \geq 1$. I described this phenomenon in my work during 1991–1993.

At the invitation of Mirek Finger (again JINR), I spoke about this phenomenon at a conference in Prague. The chairman of the section in which I presented the report was academician A. M. Baldin, who responded very positively to my work.

At the same conference, I met the German physicist Prof. Frank Rathmann, and we decided to conduct an experiment to detect the effect at the COSY accelerator in Jülich. During preparation, Prof. F. Rathmann arranged cooperation with the University of Cologne, where we managed to prepare and conduct an experiment to observe spin dichroism for deuterons with 20 MeV energy, when passing through a carbon target (particle birefringence is accompanied by spin rotation (oscillations) and spin dichroism). The experiment was carried out successfully.

After some time, Dubna scientists successfully observed spin dichroism of deuterons with a momentum of 5 GeV/ c . We also participated in the second stage of that experiment. Thus, the joint work of Minsk and Dubna led to the discovery of spin dichroism at high energies. All this opens up new possibilities for further investigations of nuclear spin precession and birefringence effects at the Nuclotron-M/NICA complex.

Thus, JINR played a colossal role in my life. Currently, further research of nuclear optics at high energies at the Nuclotron-M/NICA complex will undoubtedly lead to plenty of new results.

2. Light optics

The phenomena of interference, diffraction and refraction of light are well known even to lycee and college students. A great variety of their applications is described in school and university manuals and popular science books [1–5]. The centuries-long argument about the nature of light — whether light is a wave or a particle — finally led to the creation of quantum mechanics and extension of the wave conception to the behavior of any particles of matter. As a result, optical concepts and notions were also introduced for describing interaction of particles with matter, nuclei, and one another [6–8]. In particular, the diffraction of electrons by crystals, which are the fact natural diffraction gratings, is widely used nowadays. Neutron interferometers were designed [9]. It was found out that scattering of particles by nuclei (and by one another) is in many cases similar to scattering of light by a drop of water (the optical model of the nucleus).

The study of interaction between light and matter has shown that besides frequency and propagation direction, light waves are characterized by polarization.

The first experiment which observed a phenomenon caused by the polarization of light was carried out in 1669 by E. Bartholin, who discovered the double refraction of a light ray by Iceland spar (calcite). Today it is common knowledge that in the birefringence effect, the stationary states of light in a medium are the states with linear polarization parallel or perpendicular to the optic axis of a crystal. These states have different refractive indices and move at different velocities in a crystal. As a result, for example, circularly polarized light in crystals turns into linearly polarized and vice versa [1].

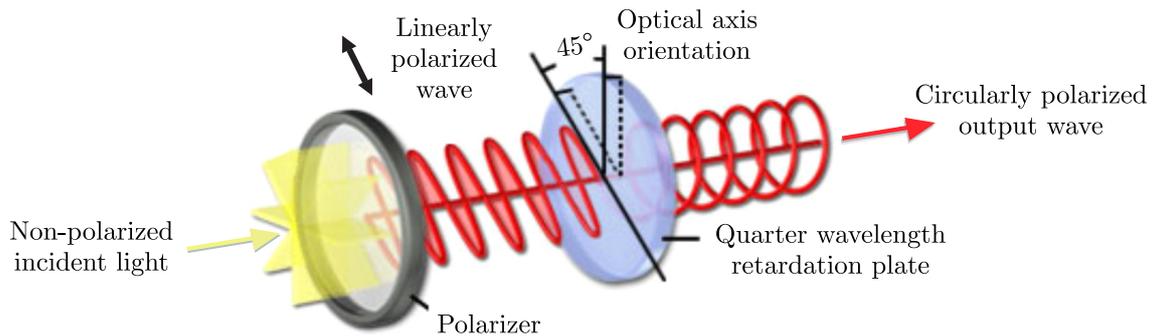


Figure 1. Light optics: birefringence effect.

Another series of experiments was performed by D. F. Arago in 1811 and J. B. Biot in 1812. They discovered the phenomenon of optical activity, in which the light polarization plane rotates as the light passes through the medium. In 1817, A. Fresnel established that in an optically active medium rotating the polarization plane, the stationary states are the waves with right-hand and left-hand circular polarizations, which, as he found out in 1823, move in a medium at different velocities (i.e., propagate with different refractive indices), thus causing the polarization plane to rotate. Let us also recall the effect of light polarization plane rotation in matter placed in a magnetic field, which was discovered by Faraday, and the birefringence effect in matter placed in an electric field (the Kerr effect).

The above-mentioned phenomena and various other effects caused by the presence of polarization of light and optical anisotropy of matter have become the subjects of intensive studies and found wide applications. The microscopic mechanism leading to the appearance of optical anisotropy of matter is, in the final analysis, due to the dependence of the process of electromagnetic wave scattering by an atom (or molecule) on the wave polarization (i.e., on the

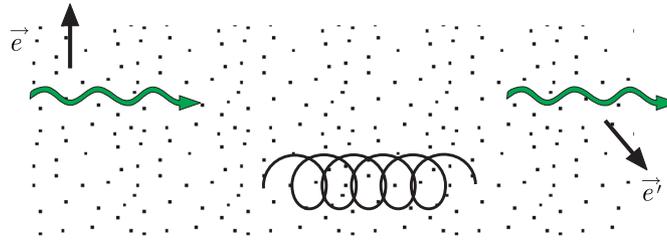


Figure 2. Light optics: natural optical activity.

photon spin) and to bounds imposed on electrons in atoms and molecules. Beyond the optical spectrum, when the photon frequency appears to be much greater than the characteristic atomic frequencies, such bounds become negligible, and the electrons can be treated as free electrons. As a result, the effects caused by optical anisotropy of matter, which are studied in optics, rapidly diminish, becoming practically unobservable when the wavelengths are smaller than 10^{-8} cm.

Moreover, there is a widespread belief that it is only possible to speak of the refraction of light and to use the concept of the refraction index of light in matter because the wavelength of light ($\lambda \approx 10^{-4}$ cm) is much greater than the distance between the atoms of matter R_a ($R_a \approx 10^{-8}$ cm), since only in this case ($\lambda \gg R_a$) matter may be treated as a certain continuous medium. As a consequence, in a short-wave spectral range where the photon wavelength is much smaller than the distance between the atoms of matter, the effects similar to the Faraday effects and birefringence, which are due to refraction, should not occur. However, such a conclusion turned out to be incorrect. The existence of refraction phenomena does not appear to be associated with the relation between the wavelength λ and the distance between atoms (between scatterers). Even at high photon energies, when the wavelength is much smaller than R_a , the effects due to refraction of waves in matter can be quite appreciable. Thus, for example, when a beam of linearly polarized γ -quanta with the energies greater than tens of kiloelectronvolts (wavelengths smaller than 10^{-9} cm) passes through matter with polarized electrons, there appears rotation of the polarization plane of γ -quanta, which is kinematically analogous to the Faraday effect ([10–14]). Moreover, with the growth of the energy of γ -quantum (the decrease in the γ -quantum wavelength) the effect increases, attaining its maximum in the megaelectronvolt energy range. Unlike the Faraday effect, which is due to the bounds imposed on electrons in atoms, for γ -quanta electrons may be treated as free electrons. The effect of polarization plane rotation in this case is due to the quantum–electrodynamic radiative corrections to the process of scattering of γ -quanta by an electron, which are lacking in classical electrodynamics.

Analogously, the propagation in matter of the de Broglie waves, which describe the motion of massive particles, can be characterized by the refractive index [7, 15]. In this case, the refractive index also characterizes particle motion in matter, even at high energies, for which the de Broglie wavelength \hbar/mv (m is the particle mass, v is its velocity; in the case of relativistic velocities, m stands for the relativistic mass $m\gamma$, γ is the Lorentz factor) is small in comparison with the distance between the atoms (scatterers). Furthermore, it turns out that for particles with nonzero spin, there exist phenomena analogous to light polarization plane rotation and birefringence. In this case, such phenomena of quasi-optical activity of matter (“optical” anisotropy of matter) are due not only to electromagnetic but also to strong and weak interactions.

The investigations in this field were initiated by V. Baryshevsky and M. Podgoretsky [16], who predicted the existence of the phenomenon of quasi-optical spin rotation of the neutron

moving in matter with polarized nuclei, which is caused by strong interactions, and introduced the concept of a nuclear pseudomagnetic field (neutron spin precession in a pseudomagnetic field of matter with polarized nuclei). The concept of a nuclear pseudomagnetic field and the phenomenon of neutron spin precession in matter with polarized nuclei were experimentally verified by Abragam's group in France (1972) (see [17] and references therein) and Forte in Italy (1973) [18].

Here, we would also mention the paper by F. Curtis Michel [19], who predicted the existence of spin "optical rotation" due to parity nonconserving weak interactions (the phenomenon was experimentally observed [20] and is used for studying parity nonconserving weak interactions between neutrons and nuclei).

Further analysis showed that the effects associated with the optical activity of matter, which we consider in optics, are, in fact, the particular case of coherent phenomena emerging when polarized particles pass through matter with nonpolarized and polarized electrons and nuclei [21–23]. It was found out, in particular, that at high energies of particles (tens, hundreds and thousands of gigaelectronvolts), the effects of "optical anisotropy" are quite significant and they may become the basis of unique methods for the investigation of the structure of elementary particles and their interactions.

3. Gamma optics

Preceding the consideration of different quasi-optical phenomena in the high-energy range, I would like to mention that, so far, the following statements, similar to those recently expressed by a reviewer of a distinguished journal, still appear: "... and the whole classical concept of a refractive index, which underpins this calculation, requires a beam of light coherent over some spatial extent much larger than the distance between the scatterers so that the (forward) scattered radiation interferes constructively to produce a phase-shifted beam — it is not a obvious that the gamma-ray beam at, say, MAMI satisfies this ..." The above misinterpretation compels me to remind that numerous studies have made it possible to establish a close relation between the elastic coherent zero-angle scattering amplitude $f(0)$ and the refractive index n (see, for example, [7, 15, 24–27]) and to develop experimental methods for investigating refraction of polarized particles in matter. It turns out that in the case when matter is composed of randomly located scatterers and the condition $|n - 1| \leq 1$ is fulfilled, the refractive index has the form

$$n = 1 + \frac{2\pi\rho}{k^2} f(0), \quad (1)$$

where ρ is the density of scatterers (the number of scatterers per cubic centimeter of matter); k is the wave number of the incident wave.

It was also found out that the possibility of introducing the refractive index is not associated with the ratio of the radiation wavelength to the distance between scatterers. (1) also describes the refraction of short-wave radiation with a wavelength much shorter than the distance between the scatterers. This can be explained by the fact that the refractive index appears due to the interference between an incident wave and secondary rescattered waves, which always occurs in elastic coherent forward scattering. Moreover, (1) describes not only the scattering of photons but also the scattering of particles of different nature (neutrons, electrons and others).

It is well known that an optically anisotropic medium is characterized by the presence of several refractive indices. For instance, in the case of the Faraday effect, the refractive indices n_+ and n_- are different for light with right-hand and left-hand circular polarizations. In view

of (1), it follows that the amplitude f_+ of elastic coherent forward scattering of a right-hand polarized photon differs from a similar amplitude f_- for a left-hand polarized photon.

Thus, for the effect of light polarization plane rotation in a medium to occur, the elastic coherent zero-angle scattering amplitude should depend on the photon polarization state, or, which is the same, on its spin state.

As spin dependence of the scattering process is typical of particle collisions for all particles with nonzero spin, it should be supposed that quasioptical phenomena analogous to the Faraday effect (birefringence) will occur for all such particles and various interactions.

It is well known that phenomena caused by the optical anisotropy of matter (the Faraday effect, birefringence, natural rotation of the light polarization plane) are eventually due to the influence that the forces acting on electrons in atoms have on the interaction of electromagnetic waves with matter.

Beyond the optical spectrum, when the frequency of electromagnetic waves becomes much greater than the average energy of electrons in atoms and molecules, the interaction between radiation and matter is reduced to the interaction of a photon with free electrons. As a result, the structure of atoms and molecules becomes non-essential, and hence the phenomena caused by the optical anisotropy of matter should disappear. For example, in the case of the Faraday effect, a simple theory based on the normal Zeeman effect gives the following expression for the light polarization plane rotation angle ϑ per 1 cm path length [28]:

$$\vartheta = \frac{\omega}{c} \frac{\partial n(\omega)}{\partial \omega} \frac{eB}{2mc}, \quad (2)$$

where $n(\omega)$ is the refractive index of matter in the absence of a magnetic field; e is the electron charge; m is its mass.

In a high-frequency range, the expression for $n(\omega)$ can be written as [28]

$$n = 1 - \frac{2\pi e^2 N}{m\omega^2},$$

where N is the number of electrons per 1 cm³ of matter, m is the electron mass. Thus, we have for the rotation angle ϑ of a high-energy quantum:

$$\vartheta = \frac{2\pi e^3 N}{m^2 c^2 \omega^2}. \quad (3)$$

From (3), we obtain that in the range of γ -quantum energies of 100 keV, the angle $\vartheta \approx 10^{-7}$ rad/cm for $B = 10^5$ Gs = 10 T, $N \approx 10^{23}$ and decreases rapidly with increasing quantum frequency. Similarly, all other magneto-optical effects — for instance, the inverse Faraday effect [29] — as well as electro-optical effects studied in optics decay rapidly and become negligibly small for gamma-quanta.

However, in 1965, V. Baryshevsky and V. Lyuboshitz showed [10] that just at high energies of γ -quanta, another mechanism of photon polarization plane rotation becomes possible in a target with polarized electrons, which is due to spin dependence of the amplitude of the elastic Compton forward scattering of a γ -quantum by an electron. The calculations showed that the rotation angle reaches its maximum value of $5.32 \cdot 10^{-3}$ rad/cm in the range of quantum energies from 500 to 700 keV [11]. This new phenomenon was experimentally observed by V. M. Lobashev, L. A. Popeko, L. M. Smotritskii, A. P. Serebrov, E. A. Kolomenskii [12, 13], by P. Bock and P. Luksch [14] and was registered as a scientific discovery in the USSR State registry under the number 360.

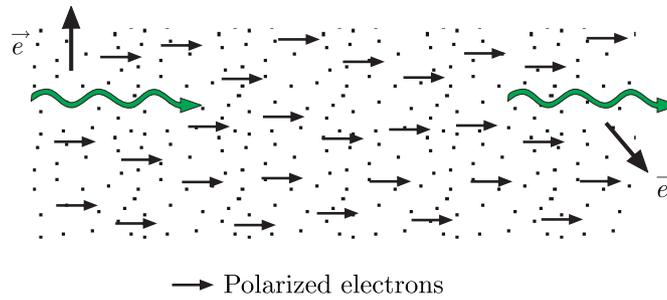


Figure 3. Gamma-quanta polarization plane rotation.

Let us now consider processes caused by γ -quanta interaction with protons and nuclei of matter. In accordance with the pioneering results obtained by M. Gell-Mann, M. L. Goldberger and W. E. Thirring [30], the amplitude of forward elastic scattering of a photon by a particle with spin $s = 1/2$ can be written as follows:

$$f_{\mu\nu}(\omega) = f_1(\omega) (\vec{e}_\mu^* \vec{e}_\nu) + i f_2(\omega) \vec{\sigma} [\vec{e}_\mu^* \vec{e}_\nu], \quad (4)$$

where \vec{e}_μ and \vec{e}_ν are the photon polarization vectors in states μ and ν , $\vec{\sigma}$ is the Pauli spin matrix.

Another notation is often used now when considering γ -quanta interaction with protons (nuclei), namely: $f_1(\omega) = f_0(\omega)$ and $f_2(\omega) = g_0(\omega)$. Therefore, for further consideration and analysis of (4), we will use it in the following form (see, for example [31–35])

$$f_{\mu\nu}(\omega) = f_0(\omega) (\vec{e}_\mu^* \vec{e}_\nu) + i g_0(\omega) \vec{\sigma} [\vec{e}_\mu^* \vec{e}_\nu]. \quad (5)$$

According to analysis [31–35], equations (4) and (5) can be used to define electromagnetic polarizabilities and spin polarizabilities as the lowest-order coefficients in an ω -dependent development of the scattering amplitudes. It was shown that the real part of amplitude $g_0(\omega)$, which determines the spin-dependent part of amplitude $f_{\mu\nu}(\omega)$ at small frequencies, can be expressed as:

$$g_0(\omega) = -\frac{e^2 \kappa^2}{8\pi m^2} \omega + \gamma_0 \omega^3, \quad (6)$$

where γ_0 is the spin polarizability, in the selected system of units electric charge e satisfies the relation $\frac{e^2}{4\pi} = \frac{1}{137.04}$, κ is the anomalous magnetic moment expressed in nuclear magneton. For a proton $\kappa = 1.79$, $\gamma_0 \sim -10^{-4} \text{ fm}^4$.

When the frequency increases, expression (6) for $g_0(\omega)$, which includes constant γ_0 , becomes invalid. Therefore, the investigation of the $g_0(\omega)$ dependence on frequency ω for high-energy γ -quanta (energy range from a few to dozens GeV or even higher) is of interest.

3.1. Quasi-optical phenomenon of γ -quanta polarization plane rotation in matter with polarized proton (nuclei)

Let us consider the passage of a beam of γ -quanta through a medium with polarized protons (nuclei, electrons).

Pioneering publications considering application of γ -quanta for the investigation of the internal structure of a nucleon appeared in the 1950s [30, 39]. These papers initiated numerous theoretical and experimental studies of this interesting possibility [10, 31–35, 38, 40–60]. The first experiment on Compton scattering by the proton to measure the polarizabilities was carried out in 1958 (see [32, 33]).

Fast advancing of experimental methods makes both single and double polarization experiments possible with a polarized solid target and polarized high-energy γ -quanta [31–35, 40–60]. Polarized targets with high degree of nuclei polarization were created, namely: NH_3 , ND_3 and ${}^6\text{LiD}$, as well as a butanol polarized proton (deuteron) target [52–56, 59, 60].

Let us intently consider the passage of a γ -quanta beam through a medium with polarized electrons (nuclei) [36]. If the photon in right- and left-circularly polarized states has different refractive indices n_1 and n_2 , then [10, 37]:

$$\Delta n = n_2 - n_1 = \frac{2\pi N c^2}{\omega^2} [f_-(0) - f_+(0)], \quad (7)$$

where N is the number of electrons (nuclei) per unit volume; $f_+(0)$ and $f_-(0)$ are the spin-non-flip amplitudes of elastic zero-angle scattering of right- and left-circularly polarized photons by polarized electrons (nuclei), respectively; ω is the photon frequency.

The scattering amplitude for the Compton forward scattering by a particle with spin 1/2 can be written as follows [30]:

$$f_{\mu\nu} = f_1(\omega)(\vec{e}_\mu^* \vec{e}_\nu) + i f_2(\omega) \vec{\sigma} [\vec{e}_\mu^* \vec{e}_\nu], \quad (8)$$

where \vec{e}_μ and \vec{e}_ν are the photon polarization vectors in states μ and ν ; $\vec{\sigma}/2$ is the particle spin operator. For the state with the right-hand circular polarization

$$\vec{e}_+ = -(\vec{e}_1 + i \vec{e}_2)/\sqrt{2}, \quad (9)$$

while for the state with the left-hand circular polarization

$$\vec{e}_- = (\vec{e}_1 - i \vec{e}_2)/\sqrt{2}, \quad (10)$$

Here $\vec{e}_2 = [\vec{e}_1 \vec{n}]$, where \vec{n} is the unit vector pointing in the propagation direction of the beam of γ -quanta.

In view of the above, it is easy to demonstrate that

$$f_+ = f_1(\omega) - f_2(\omega)(\vec{p}\vec{n}), \quad f_- = f_1(\omega) + f_2(\omega)(\vec{p}\vec{n}), \quad (11)$$

where \vec{p} is the polarization vector of particles.

Suppose that photons in a vacuum are linearly polarized along the direction \vec{e}_1 . Then, according to [10], in the medium the polarization vector \vec{e}'_1 is:

$$\begin{aligned} \vec{e}'_1 &= \left[\left(\frac{\vec{e}_1 + i \vec{e}_2}{2} \right) \exp \left(-i \frac{\omega}{2c} \Delta n l \right) \right. \\ &+ \left. \left(\frac{\vec{e}_1 - i \vec{e}_2}{2} \right) \exp \left(i \frac{\omega}{2c} \Delta n l \right) \right] \exp \left(i \frac{n_1 + n_2}{2c} \omega l \right) \\ &= \exp \left(i \omega \frac{n_1 + n_2}{c} l \right) \left[\vec{e}_1 \cos \left(\frac{2\pi N c}{\omega} (\vec{p}\vec{n}) f_2(\omega) l \right) \right. \\ &+ \left. \vec{e}_2 \sin \left(\frac{2\pi N c}{\omega} (\vec{p}\vec{n}) f_2(\omega) l \right) \right], \end{aligned} \quad (12)$$

where l is the path passed by γ -quanta in matter.

Full rotation of the polarization vector takes place in the length

$$d = \frac{4\pi c}{\omega |\Delta n|}. \quad (13)$$

One can easily see that the positive sign of $f_2(\omega)(\vec{p}\vec{n})$ corresponds to the right-hand rotation, while the negative sign of this quantity corresponds to the left-hand rotation.

In the general case, $\text{Im}f_2(\omega) \neq 0$, i.e., the coefficients of absorption are different for the states with left and right circular polarizations. As in this case (12) includes the trigonometric functions for complex arguments, the dependence of photon polarization on distance l becomes more complicated. If the photons in a vacuum are still polarized along the direction \vec{e}_1 , the following formulas are applicable to the Stokes parameters in a medium [61, 62]

$$\varepsilon_1 = r \cos 2\varphi; \quad \varepsilon_2 = (1 - r^2)^{1/2}; \quad \varepsilon_3 = r \sin 2\varphi, \quad (14)$$

where

$$\begin{aligned} \varphi &= \frac{2\pi Nc}{\omega} (\vec{p}\vec{n}) \text{Re}f_2(\omega) l; \\ r &= \cosh \left(\frac{4\pi Nc}{\omega} (\vec{p}\vec{n}) \text{Im}f_2(\omega) l \right) = (\varepsilon_1^2 + \varepsilon_3^2)^{1/2} \end{aligned}$$

is the degree of linear polarization; ε_2 is the degree of circular polarization. At $l = 0$, we have $\varepsilon_1 = 1$, $\varepsilon_3 = 0$, $\varepsilon_2 = 0$.

It is seen that when the imaginary part of the function $f_2(\omega)$ is nonzero, the linear polarization of the photon in a medium transforms into an elliptical one, and φ is the angle of rotation of the ellipse's major axis relative to the initial direction \vec{e}_1 .

It follows from the above that the full rotation of the ellipse's major axis occurs in the length

$$d = \left(\frac{Nc}{\omega} (\vec{p}\vec{n}) \text{Re}f_2(\omega) \right)^{-1}. \quad (15)$$

Note that at $l \rightarrow \infty$, we have $|\varepsilon_2| = 1$. This indicates the total absorption of photons with right- or left-hand circular polarization.

It immediately follows from (14) that the change in the polarization of γ -quanta passing through a polarized target only depends on the function $f_2(\omega)$. As for the function $f_1(\omega)$ (see (8)), it has nothing to deal with the effect we are concerned with.

From the optical theorem, the following relation follows

$$\text{Im}f_2(\omega) = \frac{\omega}{4\pi c} \frac{\sigma_{\uparrow\downarrow}(\omega) - \sigma_{\uparrow\uparrow}(\omega)}{2}, \quad (16)$$

where $\sigma_{\uparrow\uparrow}$ and $\sigma_{\uparrow\downarrow}$ are the values of the total Compton scattering cross sections for parallel and antiparallel orientations of photon and electron (nucleus) spins, respectively.

To calculate the real part of $f_2(\omega)$, we make use of the dispersion relation given in [30].

$$\text{Re}f_2(\omega) = -\frac{2\omega}{\hbar c^2} (\Delta\mu)^2 + \frac{2\omega^3}{\pi} \int_0^\infty \frac{\text{Im}f_2(\omega')}{\omega'^2(\omega'^2 - \omega^2)} d\omega', \quad (17)$$

where $\Delta\mu$ is the anomalous magnetic moment of the particle.

Using expression (16) for $\text{Im}f_2(\omega)$, one can express $\text{Re}f_2(\omega)$ as follows:

$$\text{Re}f_2(\omega) = -\frac{2\omega}{\hbar c^2} (\Delta\mu)^2 + \frac{\omega^3}{4\pi^2 c} \int_0^\infty \frac{\sigma_{\uparrow\downarrow}(\omega') - \sigma_{\uparrow\uparrow}(\omega')}{\omega'(\omega'^2 - \omega^2)} d\omega'. \quad (18)$$

According to expression (14), the polarization plane rotation angle for γ -quanta moving through a polarized target is determined by $\text{Re}f_2(\omega)$. Expression (18) for $\text{Re}f_2(\omega)$ comprises two summands, both conditioned by scattering: the first one is caused by the anomalous magnetic moment $\Delta\mu$, while the second is associated with other scattering processes and reactions caused by γ -quanta interactions with protons (nuclei, electrons).

In accordance with expressions (14) and (18), the polarization plane rotation angle φ for a γ -quantum, which passed in matter path l , reads as follows:

$$\varphi = -\frac{4\pi N}{\hbar c}(\Delta\mu)^2(\vec{p}\vec{n})l + \frac{2\pi Nc}{\omega}(\vec{p}\vec{n})l \frac{2\omega^3}{\pi} \int_0^\infty \frac{\text{Im}f_2(\omega')}{\omega'^2(\omega'^2 - \omega^2)} d\omega'. \quad (19)$$

Expression (19) can be rewritten as follows:

$$\varphi = -\frac{4\pi N}{\hbar c}(\Delta\mu)^2(\vec{p}\vec{n})l + 2\pi Nc(\vec{p}\vec{n})l\gamma_0(\omega)\omega^2, \quad (20)$$

where

$$\gamma_0(\omega) = \frac{1}{4\pi^2} \int_0^\infty \frac{\sigma_{\downarrow\uparrow}(\omega') - \sigma_{\uparrow\uparrow}(\omega')}{\omega'(\omega'^2 - \omega^2)} d\omega'. \quad (21)$$

In the low energy range, according to [32–35, 43–58], the amplitude of Compton forward scattering can be expressed in the form (6) with γ_0 read as follows:

$$\gamma_0 = \frac{1}{4\pi^2} \int_{\omega_{\text{thr}}}^\infty \frac{\sigma_{\downarrow\uparrow}(\omega') - \sigma_{\uparrow\uparrow}(\omega')}{\omega'^3} d\omega', \quad (22)$$

where ω_{thr} is the pion photoproduction threshold. Contributions to the total cross-sections, caused by Compton scattering and processes of electron–positron pair production, are not considered in (22).

Carried-out experiments and theoretical analysis provide the following evaluation for the proton spin polarizability γ_0 : $\gamma_0 \approx -1.34 \cdot 10^{-4} \text{ fm}^4 = -1.34 \cdot 10^{-56} \text{ cm}^4$. Therefore, in the case that (6) is valid, the polarization plane rotation angle φ for a γ -quantum, which passed in polarized matter path l , reads as follows:

$$\varphi = -\frac{4\pi N}{\hbar c}(\Delta\mu)^2(\vec{p}\vec{n})l + 2\pi N(\vec{p}\vec{n})\gamma_0 k^2 l = \varphi(\Delta\mu) + \varphi(\gamma_0), \quad (23)$$

where $k = \frac{\omega}{c}$ is the wavenumber of the γ -quantum.

The summand $\varphi(\Delta\mu)$, which includes anomalous magnetic moment, does not depend on energy, while another one $\varphi(\gamma_0)$, which is determined by spin polarizability γ_0 , depends on γ -quantum energy. The latter grows proportionally to k^2 , i.e., proportionally to the squared γ -quantum energy.

Let us now evaluate the value of the polarization plane rotation angle φ for a γ -quantum, which passes through a target with polarized nuclei. Target $^{14}\text{NH}_3$, which is used for the investigation of polarized γ -quanta scattering by polarized protons, can be considered as an example to evaluate the rotation effect. Note, that this target comprises both protons and nuclei ^{14}N , therefore, the number of nuclei per cm^3 , in this target is not equal to the number of protons. To evaluate the number of polarized protons per cm^3 let us use the density ρ in units $[\frac{\text{g}}{\text{cm}^3}]$, which for solid NH_3 is as high as $\rho = 0.85[\frac{\text{g}}{\text{cm}^3}]$. The weight of NH_3 molecule is with high accuracy equal to $M = 17m_p$, where m_p is the proton mass. Therefore, the number of NH_3

molecules per cm^3 reads $N_{\text{mol}} = \frac{\rho}{17m_p}$. And, since each molecule comprises three protons, the number of protons can be expressed as

$$N = 3N_{\text{mol}} = \frac{\rho}{m_p} \frac{3}{17} = \frac{\rho}{m_p} f.$$

Here f is the dilution factor (see, for example, [52, 53, 59, 60]) and $\frac{\rho}{m_p}$ is the number of nucleons per cm^3 . The complicated internal structure of the target requires the target density to be reduced by the so called [52, 53, 59, 60] packing factor $\varkappa = 0.6$ resulting in correction in the number of protons per cm^3 as follows:

$$N = \frac{\rho}{m_p} f \varkappa.$$

Therefore, the polarization plane rotation angle φ is finally expressed as follows:

$$\varphi = -\frac{4\pi}{\hbar c} \frac{\rho}{m_p} f \varkappa p (\vec{n}_p \vec{n}) (\Delta\mu)^2 l + \frac{2\pi\rho}{m_p} f \varkappa p (\vec{n}_p \vec{n}) \gamma_0 k^2 l, \quad (24)$$

where p is the proton polarization degree, \vec{n}_p is the unit vector directed along proton polarization vector.

If γ -quanta momentum is directed along the polarization vector, then $\vec{n}_p \vec{n} = +1$, in case of antiparallel directed \vec{n}_p and \vec{n} , product $\vec{n}_p \vec{n} = -1$. Therefore, change of \vec{n}_p direction with respect to \vec{n} results in change of rotation direction (sign). For $\vec{n}_p \uparrow \vec{n}$, the polarization plane rotation angle reads as

$$\varphi = -\frac{4\pi\rho\varkappa}{\hbar c m_p} f p (\Delta\mu)^2 l + \frac{2\pi\rho\varkappa}{m_p} f p \gamma_0 k^2 l. \quad (25)$$

Let us evaluate the rotation angle φ . In the case of NH_3 target with polarized protons, the number of protons per cm^3 is $N = \frac{\rho\varkappa f}{m_p} \approx 5.4 \cdot 10^{22}$; therefore, at a polarization degree $p = 0.9$, the number of polarized protons in the target is as high as $N_p \approx 5 \cdot 10^{22}$. The anomalous magnetic moment $\Delta\mu = 8.95 \cdot 10^{-24}$ erg/Gs; therefore, the contribution to the rotation angle $\varphi(\Delta\mu)$ can be evaluated as

$$\varphi(\Delta\mu) = 1.6 \cdot 10^{-6} l \text{ rad} \quad (26)$$

and for $l = 30$ cm, the angle $\varphi(\Delta\mu) \approx 5 \cdot 10^{-5}$ rad. The contribution to the rotation angle $\varphi(\Delta\mu)$ caused by anomalous magnetic moment does not depend on γ -quantum energy.

Let us now evaluate the contribution to the rotation angle $\varphi(\gamma_0)$, which is determined by the spin polarizability γ_0 and depends on γ -quantum energy. Using the second summand in (25), one can get the following evaluation for $\varphi(\gamma_0)$ in the case of NH_3 target:

$$\varphi(\gamma_0) \approx 3 \cdot 10^{-33} k^2 l. \quad (27)$$

It follows from (27) that for γ -quanta with energy 300 MeV ($k = 1.6 \cdot 10^{13} \text{ cm}^{-1}$) passing through the target of 30 cm thickness, the rotation angle $\varphi(\gamma_0)$ is as high as $\varphi(\gamma_0) = 2 \cdot 10^{-5}$ rad. If γ -quanta energy is increased up to 1 GeV (such energies are available at the Bonn accelerator facility ELSA and at the Mainz accelerator MAMI [52, 53, 59, 60]), this contribution could reach $\varphi(\gamma_0) = 2 \cdot 10^{-4}$ rad. Further increase of γ -quanta energies results in fast growth of the rotation angle: for 3 GeV γ -quanta $\varphi(\gamma_0) = 2 \cdot 10^{-3}$ rad, for 10 GeV γ -quanta $\varphi(\gamma_0) = 2 \cdot 10^{-2}$ rad and for 100 GeV it appears to be $\varphi(\gamma_0) = 1.8$ rad for the target of 30 cm thickness.

Experiments and theoretical analysis, in which the value $\gamma_0 = -1.34 \cdot 10^{-56} \text{ cm}^4$ was obtained, were carried out for γ -quanta in the energy range $\leq 1 \text{ GeV}$. The above obtained evaluation for the angle of rotation caused by the quasi-optical phenomenon of polarization plane rotation indicates a significant effect magnitude. This is especially valid for the energy range of GeV and higher. In this energy range, the γ -quantum wavelength is either comparable to or less than the electric radius of the proton, which is equal to $0.8 \cdot 10^{-13} \text{ cm}$. The amplitude of forward scattering, and thus $\gamma_0(\omega)$, may depend on the internal structure of the proton.

The question arises: what is the magnitude of γ_0 in this range? Does γ_0 begin to depend on the γ -quantum energy that leads to a gradual decrease of the rotation angle magnitude with energy growth? Such behavior of the rotation angle is observable for γ -quanta passing through matter with polarized electrons [11, 23, 37]. In this case, the drop occurs in the range of several MeV.

It seems very interesting not only to detect the quasi-optical effect of polarization plane rotation for γ -quanta in a target containing polarized protons, but also to study the possible energy dependence of the spin polarizability $\gamma_0(\omega)$.

Let us evaluate the possibility of detecting the quasi-optical effect of polarization plane rotation for γ -quanta passing through a target with polarized protons (deuterons). For this purpose, let us estimate the number of γ -quanta required to detect the effect. As indicated above, in the γ -quanta energy range up to 10 GeV, the angle of polarization plane rotation φ is much lower than 1. Due to polarization plane rotation, a component of the polarization vector of the γ -quantum in the direction orthogonal to the polarization plane of the incident beam will appear. The magnitude of this polarization component can be measured, for example, by studying beam passage through a second polarized target containing polarized nuclei with spin ≥ 1 . In such a target, the phenomenon of birefringence and dichroism, which is sensitive to linear polarization, arises according to [36–38]. Note that dichroism sensitive to the linear polarization of γ -quanta also arises during γ -quanta passage through oriented crystals [63–65]. To distinguish the signal from γ -quanta with the polarization vector lying in the scattering plane against the background of all the scattered γ -quanta, the number of γ -quanta falling on the detector N should be greater than the value of $1/\varphi^2$.

According to the above evaluations, the number of γ -quanta during the experiment for photons with energy 300 MeV is $N > \frac{1}{\varphi^2} \approx 2.5 \cdot 10^9$. For 1 GeV energy, the number of quanta $N \approx 2.5 \cdot 10^7$; at 3 GeV, the number of quanta approaches $N \approx 2.5 \cdot 10^5$; and at 10 GeV, the number of quanta is as high as $N \approx 2.5 \cdot 10^3$.

At γ -quanta energies above 3 GeV, the observation conditions are further improved. The number of quanta N falling on the detector is equal to $N = \dot{N}T$, where \dot{N} is the intensity of the photon beam ($[\dot{N} = \text{ph/s}]$), T is the observation time.

According to [66], modern accelerating facilities are capable of providing γ -ray flux as high as $\dot{N} \approx 10^{10} \text{ ph/s}$ (the High Intensity Gamma-Ray Source (HIGS), operated by the Triangle Universities Nuclear Laboratory). This Compton gamma-ray source is capable of providing nearly mono-energetic, polarized gamma-ray beams with energies ranging from 1 to 100 MeV.

The Laser Electron Photon beamline (LEPS, Japan) produces 1300–2900 MeV photon beams with fluxes $\dot{N} \approx 10^6 - 10^7 \text{ ph/s}$.

According to calculations carried out in [67], with the use of crystals at electron energy 10 GeV, one can obtain 10^{-3} γ -quanta with energy 3 GeV per electron. In this range of γ -quanta energy and higher, according to [67], the contribution from the Bethe-Heitler radiation mechanism becomes important. This means that even at a current of $1 \text{ nA} = 10^{10} \text{ e/s}$ one can obtain $\dot{N} \approx 10^7 \text{ ph/s}$. Therefore, in the range of γ -quanta energy about 3 GeV and above

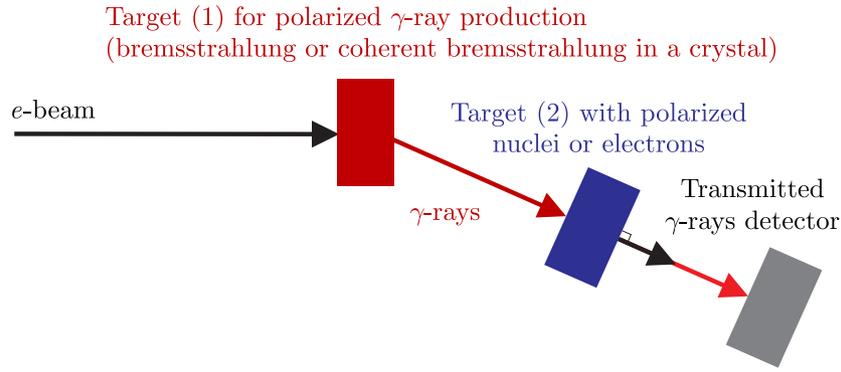


Figure 4. Layout of transmission experiment.

at SLAC accelerator complexes, it is possible to study the dependence of polarization plane rotation effect on the γ -quanta energy.

Thus, law (6) can be verified and the range for which dependence of amplitude $\gamma_0(\omega)$ on γ -quanta energy should be taken into account, can be defined.

Let us emphasize that the investigation of the quasi-optical effect of γ -quanta polarization plane rotation does not imply measurement of properties of scattered γ -quanta (scattered electromagnetic wave). The considered effect is studied by investigating the coherent passing of γ -quanta (electromagnetic wave) through a target (see Fig. 4).

The number of γ -quanta passed through the target appears to be much higher compared with the number of those scattered into some spatial angle.

It is interesting to note that the phenomenon, deceptively similar to the inverse Faraday effect [29], but caused by another mechanism [10], also appears at collisions of γ -quanta with the rest matter or a particle beam, namely: when γ -quanta with circular polarization fall onto nonpolarized matter, the latter acquires polarization due to the dependence of scattering cross-sections on the orientation of electron (nuclear) spins. The same happens when circularly polarized γ -quanta collide with a beam of nonpolarized particles, for example, protons moving in an accelerator (or a storage ring): the particle beam acquires polarization, i.e., is magnetized (the orientation of spins causes orienting magnetic moments).

3.2. Quasi-optical birefringence effect for γ -quanta in matter with polarized nuclei

It is interesting to spot that just as light in uniaxial and biaxial crystals experiences double refraction (birefringence), the similar quasi-optical birefringence effect exists for γ -quanta in matter containing polarized nuclei with spin to $S \geq 1$ [37, 38]. For example, for ^{14}N and D spin is equal $S = 1$, while ^7Li has $S = 3/2$.

According to the above (see (1)), the refractive index of matter for γ -quanta is determined by the coherent elastic forward scattering amplitude $f(0)$ as follows:

$$n = 1 + \frac{2\pi\rho}{k^2} f(0).$$

When γ -quanta interact with matter, which comprises polarized nuclei with spin $S \geq 1$, amplitude $f(0)$ can be expressed as [37, 38]:

$$f(0) = f_1(\omega)(\vec{e}'^* \vec{e}) + i f_2(\omega) \vec{p} [\vec{e}'^* \vec{e}] + f_3(\omega) Q_{ik} e_i'^* e_k + f_4(\omega) n_{\gamma i} n_{\gamma k} Q_{ik}. \quad (28)$$

where $\vec{p} = \text{Tr} \hat{\rho} \vec{n}_S$ is the nuclear polarization vector, $\vec{n}_S = \hat{S}/S$, \hat{S} is the nuclear spin operator, $\hat{\rho}$ is the spin density matrix of the target, $Q_{ik} = \text{Tr} \hat{\rho} \hat{Q}_{ik}$ is the polarization tensor of rank two,

$$\hat{Q}_{ik} = \frac{3}{2S(2S-1)} \left\{ \hat{S}_i \hat{S}_k + \hat{S}_k \hat{S}_i - \frac{2}{3} S(S+1) \delta_{ik} \right\};$$

\vec{n}_γ is the unit vector along the γ -quanta momentum. Note that amplitudes f_1 , f_2 and f_3 are expressed via polarizabilities α_0 , α_γ and α_T of nuclei introduced by A. M. Baldin [40, 41] as follows:

$$f_1 = \left(\frac{\omega}{c}\right)^2 \alpha_0, \quad f_2 = \left(\frac{\omega}{c}\right)^2 \alpha_\gamma, \quad f_3 = \left(\frac{\omega}{c}\right)^2 \alpha_T. \quad (29)$$

Let us assume that the polarization vector \vec{p} for a target is orthogonal to the γ -quanta incidence direction: $\vec{p} \perp \vec{n}_\gamma$, and define the direction of \vec{n}_γ as axis y . In this case, from (28) one could observe the difference in refraction indices for a photon with linear polarization \vec{e}_x and that with linear polarization \vec{e}_z .

Let the linear polarization of a γ -quantum incident on a target be a superposition $\vec{e} = \alpha \vec{e}_x + \beta \vec{e}_z$. As the γ -quantum moves deeper into the target, its linear polarization converts into elliptical one that is in full similarity to optics. Thus, circular polarization appears in the initially linearly polarized beam of γ -quanta. The polarization degree is determined by $\text{Re } f_3(\omega)$.

In the case when a γ -quantum with circular polarization moves in the target, it attains linear polarization, the degree of which is determined by $\text{Re } f_3(\omega)$. The detailed description of birefringence for γ -quanta is given in [37, 38].

According to evaluations [38] in the vicinity of giant resonance, the degree δ of attained circular polarization for a γ -quantum, which initially has linear polarization, when it passes through a target with $l = 1$ cm thickness comprising Ta nuclei, is as high as $\delta \approx 10^{-3} - 10^{-4}$. For a target comprising polarized deuteron nuclei, the polarization degree could be evaluated as $\delta \approx 4 \cdot 10^{-5} l$, assuming that γ -quanta energy is about a few MeV and the deuteron tensor polarizability is evaluated by its static polarizability $\alpha_T \approx 10^{-40} \text{ cm}^3$, derived in [68].

For other nuclei and γ -quanta energies as high as dozens and hundreds of MeV and higher, the birefringence effect should be evaluated separately.

Success in the development of targets with polarized nuclei and beams of polarized high-energy γ -quanta enables direct experimental observation of the quasi-optical phenomenon of γ -quanta polarization plane rotation in matter with polarized proton (nuclei). Quasi-optical birefringence effect for γ -quanta in matter with polarized nuclei having spin $S \geq 1$ could also be observed [23, 37, 38]. The latter effect is similar to double refraction (birefringence) of light in uniaxial and biaxial crystals (see Fig.5).

The above discussed experiments can be carried out at LINAC (JINR).

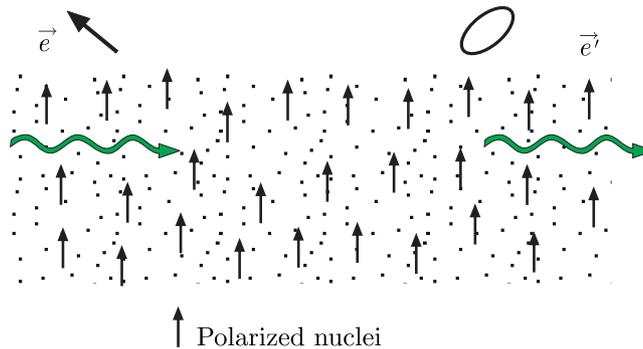


Figure 5. Birefringence of gamma quanta.

4. Quasi-optical phenomena for low-energy particles

4.1. Neutron Spin Precession in a Pseudomagnetic Field of Matter with Polarized Nuclei

When a neutron moves in matter with polarized nuclei, the phenomenon of neutron spin precession (rotation) arises. The spin rotation is caused not merely by Larmor precession in a magnetic field, but also by neutron spin interaction with the nuclear pseudomagnetic field caused by strong (weak) interaction of neutrons with polarized nuclei.

We shall start the description of the phenomenon of “optical” spin rotation of particles in matter with polarized nuclei by considering the passage of slow neutrons through matter with polarized nuclei. The possible contribution to neutron scattering from the interaction between the neutron magnetic moment and the magnetic moments of nuclei (and electrons) will be temporarily ignored. In this case, the wave describing the process of elastic collision between a neutron and a nucleus fixed at point R_i has the form

$$\psi(\vec{r}) = e^{i\vec{k}\vec{r}}\chi_n\chi_{\text{nuc}} + \hat{f} \frac{e^{ik|\vec{r}-\vec{R}_i|}}{|\vec{r}-\vec{R}_i|} e^{i\vec{k}\vec{R}_i}\chi_n\chi_{\text{nuc}}, \quad (30)$$

where χ_n is the spin wave function of the incident neutron; χ_{nuc} is the spin wave function of the nucleus.

For slow neutrons, the wavelength is much larger than the size of the nuclei. That is why the scattering amplitude \hat{f} does not depend on the angle (S -scattering) and can be written as follows [27]:

$$\hat{f} = \alpha + \beta\vec{\sigma}\vec{J}, \quad (31)$$

where

$$\alpha = \frac{J+1}{2J+1}a^+ + \frac{J}{2J+1}a^-,$$

$$\beta = \frac{a^+ - a^-}{2J+1},$$

a^+ is the scattering amplitude in the state with the total momentum $J+1/2$ of the neutron and the nucleus; a^- is the same in state $J-1/2$; $\vec{\sigma} = 2\vec{S}$, \vec{S} is the neutron spin operator; \vec{J} is the nuclear spin operator.

At scattering by a set of nuclei, the wave function takes the form

$$\psi(\vec{r}) = e^{i\vec{k}\vec{r}}\chi_n \prod_m \chi_{\text{nuc}m} + \sum_i \hat{f}_i \frac{e^{ik|\vec{r}-\vec{R}_i|}}{|\vec{r}-\vec{R}_i|} e^{i\vec{k}\vec{R}_i}\chi_n \prod_m \chi_{\text{nuc}m}, \quad (32)$$

where $\prod_m \chi_{\text{nuc}m}$ is the spin wave function of nuclei; the nuclei are assumed not to interact with each other.

To find the coherent wave in this case, (32) should be averaged over the location of scatterers, as well as over their spin state. Averaging of (32) over the spin state of the nuclei gives

$$\langle\psi(\vec{r})\rangle = e^{i\vec{k}\vec{r}}\chi_n + \sum_i \langle\hat{f}\rangle \frac{e^{ik|\vec{r}-\vec{R}_i|}}{|\vec{r}-\vec{R}_i|} e^{i\vec{k}\vec{R}_i}\chi_n, \quad (33)$$

where

$$\langle\hat{f}\rangle = \alpha + \beta\vec{\sigma}\langle\vec{J}\rangle = \alpha + \beta J\vec{\sigma}\vec{p},$$

$\vec{p} = \langle\vec{J}\rangle/J$ is the polarization vector of the nuclei.

If the nuclei are chaotically distributed in a certain plane $z = z_0$, we obtain the following expression for a coherent wave which has passed through the given plane

$$\langle \psi(\vec{r}) \rangle = \left(1 + \frac{2\pi i \rho'}{k_z} (\alpha + \beta J \vec{\sigma} \vec{p}) \right) e^{i\vec{k}\vec{r}} \chi_n, \quad (34)$$

Note that the distance between scatterers was not assumed to be less than the wavelength in when deriving (34). Expression (34) is also valid in the case when the wavelength is much smaller than the distance between scatterers.

Recall (see, for instance, [62]) that the operator of the form

$$B = 1 + \frac{i}{2} \delta\theta \vec{\sigma} \vec{\eta}_p \quad (35)$$

is the operator of spin rotation by an angle $\delta\theta$ ($\delta\theta \ll 1$) about the axis characterized by a unit vector $\vec{\eta}_p$. Comparing (35) with (34), one may conclude that after the neutron has passed the polarized plane, the neutron spin will rotate by the angle

$$\delta\theta = \frac{4\pi \rho'}{k_z} J p \text{Re} \beta. \quad (36)$$

If the wave passes through m planes, the total angle is

$$\delta\theta = \frac{4\pi \rho'}{k_z} J p m \text{Re} \beta \quad (37)$$

or, proceeding to a layer of finite thickness l , we obtain that as a neutron passes through a polarized target, its spin rotates [16] through an angle (see Fig. 6)

$$\theta = \frac{4\pi \rho}{k_z} \text{Re} \beta J p l. \quad (38)$$

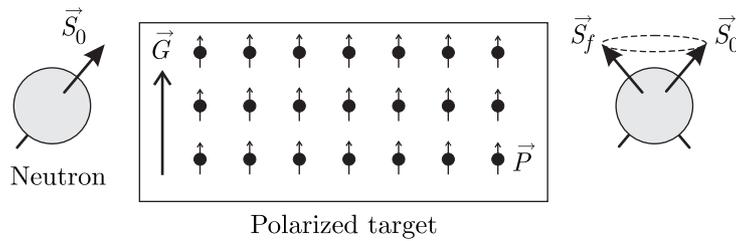


Figure 6. Neutron spin precession.

The same result can be obtained in a different way. Let us choose the quantization axis parallel to vector \vec{p} . As a result, if a neutron with spin parallel to vector \vec{p} [$\chi_n = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$] falls on the plane, the coherent wave $\langle \psi(\vec{r}) \rangle$ has the form

$$\langle \psi(\vec{r}) \rangle_+ = \left(1 + \frac{2\pi i \rho'}{k_z} f_+ \right) e^{i\vec{k}\vec{r}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (39)$$

where $f_+ = \alpha + \beta J p$ is the amplitude of coherent elastic zero-angle scattering of the neutron with spin parallel to the polarization vector of nuclei \vec{p} . For a neutron with the opposite spin direction [$\chi_n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$], the coherent wave $\langle \psi(\vec{r}) \rangle$ is given by the expression

$$\langle \psi(\vec{r}) \rangle_- = \left(1 + \frac{2\pi i \rho'}{k_z} f_- \right) e^{i\vec{k}\vec{r}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (40)$$

where $f_- = \alpha - \beta Jp$ is the corresponding coherent amplitude of scattering of the neutron with spin antiparallel to vector \vec{p} .

If a wave passes through a finite thickness layer of polarized matter, then, recalling the conclusions that led us to the expression describing the refractive index for a nonpolarized target, we obtain that the refractive index for neutrons with spin parallel to \vec{p} is

$$n_+ = 1 + \frac{2\pi\rho}{k_z^2} f_+ = 1 + \frac{2\pi\rho}{k_z^2} (\alpha + \beta Jp). \quad (41)$$

For neutrons with the opposite polarization,

$$n_- = 1 + \frac{2\pi\rho}{k_z^2} f_- = 1 + \frac{2\pi\rho}{k_z^2} (\alpha - \beta Jp). \quad (42)$$

In operator form, the expression for \hat{n} can be written as follows [22, 23] (see also (1)):

$$\hat{n} = 1 + \frac{2\pi\rho}{k_z^2} (\alpha + \beta J\vec{\sigma}\vec{p}) = 1 + \frac{2\pi\rho}{k_z^2} \hat{f}(0).$$

If k_z decreases and the wavelength increases, then (with nuclei of different kinds present in the target) the following expression is true for \hat{n}

$$\hat{n}^2 = 1 + \frac{4\pi}{k_z^2} \sum_i \rho_i \hat{f}_i(0). \quad (43)$$

The difference

$$\Delta = n_+ - n_- = \frac{2\pi\rho}{k_z^2} (f_+ - f_-) = \frac{4\pi\rho}{k_z^2} \beta Jp \quad (44)$$

is determined by the difference of the corresponding coherent scattering amplitudes and is nonzero only in a polarized medium.

Thus, in a polarized nuclear target, neutrons have two indices of refraction.

Let now neutrons with the polarization vector oriented at a certain angle to the direction of \vec{p} be incident on a polarized medium. This neutron state can be considered as a superposition of two states with polarizations along and opposite to the direction of vector \vec{p} . The wave function of a particle before entering the target has the form

$$\psi(\vec{r}) = e^{i\vec{k}\vec{r}} \chi_n, \chi_n = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \quad (45)$$

or

$$\psi(\vec{r}) = c_1 e^{i\vec{k}\vec{r}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{i\vec{k}\vec{r}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (46)$$

To be more specific, let us study the case of refraction by the target whose nuclei have the polarization vector \vec{p} directed perpendicular to its surface. Choose the direction \vec{p} as the z -axis. The state of the type $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ has the refractive index n_+ , while the state of the type $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ has the refractive index n_- . If a neutron with spin parallel to the polarization vector \vec{p} (spin state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$) were incident on the target, its motion in matter would be described by the wave function $\psi_+(r) = e^{i\vec{k}_\perp \vec{r}_\perp} e^{ik_z n_+ z} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (in this representation, we neglect the influence of mirror reflection on the wave function of the neutron passing through matter, which is possible as $|n_\pm - 1| \ll 1$ in a wide range of angles of incidence of thermal neutrons on the target).

If a neutron with spin antiparallel to \vec{p} (spin state $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$) is incident on the target, then in matter it is described by the wave function $\psi_-(r) = e^{i\vec{k}_\perp \vec{r}_\perp} e^{ik_z n - z} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. If a neutron with an arbitrary spin direction falls on the target, its wave function [see (46)] is the superposition of states $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. As a consequence, the wave function of neutrons in matter is also the superposition of these states, and can be written as

$$\psi(\vec{r}) = \begin{pmatrix} c_1 & \psi_+ & (\vec{r}) \\ c_2 & \psi_- & (\vec{r}) \end{pmatrix} = c_1 e^{i\vec{k}_\perp \vec{r}_\perp} e^{ik_z n + z} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2 e^{i\vec{k}_\perp \vec{r}_\perp} e^{ik_z n - z} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (47)$$

Now let us consider how the polarization of neutrons changes as they penetrate into the interior of the target (with the growth of the target thickness). Suppose we have a detector that transmits the particles with spin polarized along a certain direction in the detector (the axis of the detector) and absorbs the particles with the opposite spin direction. Such a detector is the analog of the Nicol prism [1] used in optics for analyzing the polarization of light. When polarized light is incident on the Nicol prism, one component of the light polarization passes through it, while the component orthogonal to the axis of the Nicol prism is absorbed. In the case of neutrons, a target with polarized nuclei may act as a detector. As the scattering cross section of a polarized neutron depends on whether the neutron spin is oriented along the direction of the polarization vector of the nucleus or opposite to it, neutron absorption in the detector exhibits the same dependence. Suppose that the axis of the detector is parallel to the z -axis along which the target nuclei are polarized. In this case, the detector analyzes those components of the neutron spin which are directed along the z -axis and opposite to it. From (47) it follows that the probability amplitude $A^{(+)}$ of finding the neutron with spin state $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, i.e., of finding the neutron polarized parallel to the z -axis, is given by the expression

$$A^{(+)} = (1 \ 0)\psi = c_1 e^{i\vec{k}_\perp \vec{r}_\perp} e^{ik_z n + z},$$

thus, the probability to find the neutron polarized parallel to the z -axis reads as follows:

$$P_z^{(+)} = |(1 \ 0)\psi|^2 = |c_1|^2 e^{-2k_z \text{Im}n + z} = |c_1|^2 e^{-\rho\sigma + z}.$$

Similarly, the probability $P_z^{(-)}$ of finding the neutron polarized opposite to the z -axis is

$$P_z^{(-)} = |(0 \ 1)\psi|^2 = |c_2|^2 e^{-2k_z \text{Im}n - z} = |c_2|^2 e^{-\rho\sigma - z},$$

where σ_\pm is the total cross section of scattering by the nucleus of the neutron polarized parallel (antiparallel) to the polarization vector of the nucleus.

Since in polarized matter $\text{Im}n_+ \neq \text{Im}n_-$ ($\sigma_+ \neq \sigma_-$), one of the components of the neutron spin wave function decays faster and at some depth the rapidly damped component may be neglected. The beam will appear polarized along the z -axis (along the direction of the polarization vector of nuclei).

Let us now rotate the detector so that its polarization axis becomes perpendicular to the direction of the target polarization. Choose the direction of the polarization axis of the detector as the x -axis.

Now the detector analyzes those components of the neutron spin which are directed along and opposite to the x -axis.

To determine the probability $P_x^{(\pm)}$ of finding the component of the neutron spin parallel (antiparallel) to the direction of the x -axis, one should expand the wave function (47) in terms

of the spin wave functions χ_x^\pm , which are the eigenfunctions of operator \hat{S}_x of the spin projection onto the x -axis. They have the form

$$\chi_x^\pm = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix}.$$

As a result, we find that the probabilities $P_x^{(\pm)}$ of neutron spin polarization along and opposite to the x -axis change with z as [16]:

$$\begin{aligned} P_x^{(+)} &= \frac{1}{2} \left\{ |c_2|^2 e^{-2k_z \text{Im}n_+ z} + |c_2|^2 e^{-2k_z \text{Im}n_- z} \right. \\ &\quad \left. + 2|c_1|c_2 |e^{-k_z \text{Im}(n_+ + n_-)z} \cos [k_z \text{Re}(n_+ - n_-)z + \delta] \right\}, \\ P_x^{(-)} &= \frac{1}{2} \left\{ |c_1|^2 e^{-2k_z \text{Im}n_+ z} + |c_2|^2 e^{-2k_z \text{Im}n_- z} \right. \\ &\quad \left. - 2|c_1|c_2 |e^{-k_z \text{Im}(n_+ + n_-)z} \cos [k_z \text{Re}(n_+ - n_-)z + \delta] \right\}, \end{aligned} \quad (48)$$

where $\delta = \delta_1 - \delta_2$ is the difference of the initial phases of states with neutron spin polarization along and opposite to the z -axis ($c_1 = |c_1|e^{i\delta_1}$; $c_2 = |c_2|e^{i\delta_2}$).

If at $z = 0$, the neutron is polarized along x , i.e.,

$$c_1 = c_2 = \frac{1}{\sqrt{2}}, \quad \delta = 0,$$

then with growing z the polarization opposite to x appears and further change of the polarization acquires the character of oscillations.

As the neutrons pass through the target, one of the components decays more strongly and the neutron beam eventually becomes polarized along or opposite to the z -axis. When a beam polarized along the z -axis is incident onto the target, no oscillations emerge: only damping occurs.

Using (47), one can find the neutron polarization vector

$$\vec{p}_n = \frac{\langle \psi | \vec{\sigma} | \psi \rangle}{\langle \psi | \psi \rangle}. \quad (49)$$

As a result,

$$\begin{aligned} p_{nx} &= 2\text{Re}c_1^* c_2 \psi_+^* \psi_- \langle \psi | \psi \rangle^{-1} \\ p_{ny} &= 2\text{Im}c_1^* c_2 \psi_+^* \psi_- \langle \psi | \psi \rangle^{-1}, \\ p_{nz} &= (|c_1 \psi_+|^2 - |c_2 \psi_-|^2) \langle \psi | \psi \rangle^{-1}. \end{aligned} \quad (50)$$

Suppose that neutron spin in a vacuum is directed perpendicular to the polarization vector of the nuclei. Choose this direction as the x -axis. In this case

$$c_1 = c_2 = 1/\sqrt{2}.$$

Using relations (50), we obtain

$$\begin{aligned} p_{nx} &= \cos[k_z \text{Re}(n_+ - n_-)z] e^{-k_z \text{Im}(n_+ + n_-)z} \langle \psi | \psi \rangle^{-1}, \\ p_{ny} &= -\sin[k_z \text{Re}(n_+ - n_-)z] e^{-k_z \text{Im}(n_+ + n_-)z} \langle \psi | \psi \rangle^{-1}, \\ p_{nz} &= \frac{1}{2} (e^{-2k_z \text{Im}n_+ z} - e^{2k_z \text{Im}n_- z}) \langle \psi | \psi \rangle^{-1}, \\ p_{nx}^2 + p_{ny}^2 + p_{nz}^2 &= 1. \end{aligned} \quad (51)$$

According to (51), as the neutron penetrates into the interior of the target, its polarization vector rotates about the polarization vector of nuclei through the angle

$$\theta = k_z \text{Re}(n_+ - n_-)z = \frac{2\pi\rho}{k_z} \text{Re}(f_+ - f_-)z, \quad (52)$$

which agrees perfectly with the result obtained above (see (38)).

At the same time, as the neutrons pass through polarized matter, the transverse components p_{nx} and p_{ny} of the polarization vector decay because neutron absorption depends on spin orientation, and finally the beam appears to be polarized along or opposite to the z -axis.

Thus, the dependence of the absorption of neutrons in the target on the orientation of their spin results in the fact that the polarization vector \vec{p}_n (recall that $|\vec{p}_n| = 1$) not only rotates about the z -axis (about the direction of the polarization vector of nuclei) but also undergoes additional rotation in the direction of the z -axis (the end point of the polarization vector moves along the unit sphere).

If the dependence of absorption on spin orientation can be neglected, the polarization vector rotates about the direction of the polarization vector of nuclei only in the (x, y) plane.

In terms of kinematics, this phenomenon is analogous to the light polarization plane rotation in a magnetic field (the Faraday effect), while spin oscillations along and opposite to the direction of the x -axis are analogous to the transitions $K^0 \rightleftharpoons \bar{K}^0$ occurring in regeneration of neutral K -mesons (see e. g. [69]).

The conclusion about neutron spin rotation in a polarized target can be derived from other considerations.

Since in a polarized nuclear target a neutron wave has two refractive indices, according to (32), in such a target it has two possible potential interaction energies U_{\pm} :

$$U_{\pm} = \frac{\hbar^2 k_z^2}{2m} (1 - n_{\pm}^2) = -\frac{2\pi\hbar^2}{m} \rho f_{\pm}(0) \quad (53)$$

or in operator form

$$\hat{U} = \frac{\hbar^2 k_z^2}{2m} (1 - \hat{n}^2) = -\frac{2\pi\hbar^2 \rho}{m} \hat{f}(0) = -\frac{2\pi\hbar^2 \rho}{m} (\alpha + \beta J \vec{\sigma} \vec{p}). \quad (54)$$

Recall now that the expression for the energy W of interaction between neutrons and a magnetic field \vec{B} [6] is similar in form to the second term in (54), i.e., $W = -\mu \vec{\sigma} \vec{B}$ (μ is the neutron magnetic moment). In this case, the interaction energy W_+ of a particle with spin parallel to \vec{B} is given by a well-known expression $W_+ = -\mu B$; the analogous quantity for a particle with the opposite spin direction, by the expression $W_- = \mu B$. The nonzero difference $W_+ - W_- = -2\mu B$ leads to the Larmor precession with frequency $\omega_L = 2\mu B/\hbar$ of neutron spin in a magnetic field \vec{B} .

Knowing the frequency ω_L , one can find the magnetic field $B = \hbar\omega_L/2\mu$.

In perfect analogy with spin behavior in a magnetic field, the non-zero difference $U_+ - U_-$ leads to the neutron spin precession about the direction of the nuclear polarization vector with the frequency [16]:

$$\omega = \left| \text{Re} \frac{U_+ - U_-}{\hbar} \right| = \frac{4\pi\hbar\rho}{m} \text{Re} \beta J p. \quad (55)$$

During time t , spin will rotate by the angle $\vartheta = \omega t$. If a polarized target has a layer of thickness l , the neutron, which enters the area occupied by the target, passes through the layer in time $t = l/v_z$.

Therefore its spin rotates through the angle

$$\vartheta = \omega \frac{l}{v_z} = \frac{m\omega l}{\hbar k_z} = \frac{2\pi\rho}{k_z} \text{Re}\beta Jpl,$$

which is in complete agreement with the result obtained above.

Continuing by analogy with the magnetic field, to describe neutron spin precession caused by nuclear interaction (hereinafter referred to as “neutron nuclear precession”), it is natural to introduce the effective magnetic field

$$B_{\text{eff}} = \hbar\omega/(2\mu), \quad (56)$$

which causes precession with the same frequency ω as an ordinary magnetic field.

As a result, we can write

$$\hat{U} = u_0 - \mu\vec{\sigma}\vec{B}_{\text{eff}}, \quad (57)$$

where

$$u_0 = -\frac{2\pi\hbar^2\rho}{m}\alpha, \quad \vec{B}_{\text{eff}} = \frac{2\pi\hbar^2\rho}{\mu m}\beta J\vec{p}. \quad (58)$$

Note that in the range of neutron energies where the scattering amplitude is constant, frequency ω is also constant and characterizes the rotatory power of matter due to nuclear interaction. This occurs when the neutron energy is small. With increasing neutron velocity, the spin precession frequency becomes energy-dependent: in particular, it gives a sharp rise near each of the resonances and can reverse sign when the neutron energy changes near the resonance because the sign of the real part of the scattering amplitude changes. It should be recalled in this connection (see e.g. [6, 27]) that in the vicinity of the resonance, the scattering amplitude is

$$f \sim \frac{1}{E - E_0 - i\Gamma/2},$$

where E is the particle energy; E_0 is the resonance energy, Γ is the width of the resonance level.

In view of (56), at low energies the value of the effective quasi-magnetic field of nuclear origin is a constant defining the matter, while at higher energies it is energy-dependent. For a polarized proton target, for instance, in the case of full polarization, $\omega = 5 \cdot 10^8 \text{ s}^{-1}$, $B_{\text{eff}} \approx 3 \cdot 10^4 \text{ Gs} = 3 \text{ T}$ and exceeds by two orders of magnitude the ordinary magnetic field created by polarized magnetic moments of protons. Under the same conditions, for thermal neutrons $v = 2.2 \cdot 10^5 \text{ cm} \cdot \text{s}^{-1}$ and the full rotation of spin occurs at length $L \approx 10^{-3} \text{ cm}$.

Nuclear spin precession phenomenon was observed in the experiments carried out by Abraham [17] and Forte [18, 20].

4.2. Growth of Nuclear Spin Precession Frequency of Antiprotons (Negative Hyperons) Under Deceleration in Matter with Polarized Nuclei

The progress in the development of the Facility for Low-Energy Antiproton and Ion Research (FLAIR) has spurred the rapid development of low-energy antiproton physics [70, 71].

The possibility to obtain polarized antiprotons by a spin-filtering method [72] opens up opportunities for investigating a large number of spin-dependent fundamental processes arising when antiprotons pass through matter with polarized nuclei. In particular, the study of the phenomenon of particle “optical” spin rotation in matter with polarized nuclei enables investigation of the spin-dependent part of the forward scattering amplitude [37, 73–75].

For low-energy neutrons, the phenomenon of “optical” spin rotation (the phenomenon of nuclear precession of the neutron spin in a nuclear pseudomagnetic field of a polarized target) was predicted in [16] and experimentally observed in [17, 18]. Based on this phenomenon, the spin-dependent forward scattering amplitude of thermal neutrons has been measured [76, 77], and similar new experiments are being prepared [78–81].

In contrast to neutrons, a charged particle undergoes Coulomb interaction with the atoms of matter, which causes multiple scattering and rapid deceleration of the charged particle due to ionization energy losses. With decreasing particle energy, the influence of the Coulomb interaction on particle scattering by the nucleus grows in significance. In particular, when the energy of a positively charged particle diminishes, the Coulomb repulsion suppresses nuclear interaction between the incident particle and the target nucleus and hence, the phenomenon of spin rotation due to nuclear interaction. Conversely, a negatively charged particle (antiproton, hyperon) is attracted to the nucleus and, as a result, participates in nuclear interaction even at low energies. As a consequence of this, spin rotation of a negatively charged particle in polarized matter does not disappear at very low energies either.

The present section considers the influence of the Coulomb interaction on the phenomenon of “optical” spin rotation of negatively charged particles moving in matter with polarized nuclei. It is shown that because the density of the antiproton (negative hyperon) wave function on the nucleus increases, the spin precession frequency grows as the particle decelerates. As a result, spin rotation of negatively charged particles becomes observable despite their rapid deceleration. This provides information about the spin-dependent part of the scattering amplitude in the range of low energies, where scattering experiments are practically impossible to perform.

4.3. Forward Scattering Amplitude of Negatively Charged Particles

According to [16, 37, 73–75], the spin rotation frequency Ω_{nuc} of a nonrelativistic particle passing through a target with polarized nuclei can be expressed as

$$\Omega_{\text{nuc}} = \frac{\Delta \text{Re} U_{\text{eff}}}{\hbar} = \frac{2\pi\hbar}{m} N P_t \Delta \text{Re} f(0), \quad (59)$$

where $\Delta \text{Re} U_{\text{eff}}$ is the difference between the real parts of the effective potential energy of interaction between the particle and the polarized target for oppositely directed particle spins, m is the mass of the particle, N is the number of nuclei in 1 cm^3 , P_t is the degree of polarization of the target nuclei, and $\Delta \text{Re} f(0)$ is the difference between the real parts of the amplitudes of coherent forward scattering for particles with oppositely directed spins.

The scattering amplitude $f(0)$ is related to the \mathcal{T} -matrix as follows (see, e.g. [15, 82]):

$$f(0) = -\frac{m}{2\pi\hbar^2} \langle \Phi_a | \mathcal{T} | \Phi_a \rangle, \quad (60)$$

where $|\Phi_a\rangle$ is the wave function describing the initial state of the system “incident particle–atom (nucleus)”. The wave function $|\Phi_a\rangle$ is the eigenfunction of the Hamiltonian $\hat{H}_0 = H_p(\vec{r}_p) + H_A(\vec{\xi}, \vec{r}_{\text{nuc}})$, i.e., $\hat{H}_0|\Phi_a\rangle = E_a|\Phi_a\rangle$; $H_p(\vec{r}_p)$ is the Hamiltonian of the particle incident onto the target; \vec{r}_p is the particle coordinate; $H_A(\vec{\xi}, \vec{r}_{\text{nuc}})$ is the atomic Hamiltonian; $\vec{\xi}$ is the set of coordinates of the atomic electron; \vec{r}_{nuc} is the set of coordinates describing the atomic nuclei.

The Hamiltonian H describing the particle–nucleus interaction can be written as:

$$H = H_0 + V_{\text{Coul}}(\vec{r}_p, \vec{\xi}, \vec{r}_{\text{nuc}}) + V_{\text{nuc}}(\vec{r}_p, \vec{r}_{\text{nuc}}), \quad (61)$$

where V_{Coul} is the energy of Coulomb interaction between the particle and the atom, and V_{nuc} is the energy of nuclear interaction between the particle and the atomic nucleus.

According to the quantum theory of reactions [15, 82], in the case of two interactions, the matrix element of the operator \mathcal{T} , which describes the system transition from the initial state $|\Phi_a\rangle$ into the final state $|\Phi_b\rangle$, can be presented as a sum of two terms:

$$\mathcal{T}_{ba} = \mathcal{T}_{ba}^{\text{Coul}} + \mathcal{T}_{ba}^{\text{nuc Coul}} = \langle \Phi_b | \mathcal{T}_{\text{Coul}} | \Phi_a \rangle + \langle \varphi_b^{(-)} | \mathcal{T}_{\text{nuc}} | \varphi_a^{(+)} \rangle, \quad (62)$$

where the first term, $\mathcal{T}_{ba}^{\text{Coul}}$, describes the contribution to the T-matrix that comes from the Coulomb scattering alone, the operator

$$\mathcal{T}_{\text{Coul}} = V_{\text{Coul}} + V_{\text{Coul}}(E_a - H_0 + i\varepsilon)^{-1}\mathcal{T}_{\text{Coul}}, \quad (63)$$

and the second term describes the contribution to the T-matrix that comes from nuclear scattering and accounts for the distortion of waves incident onto the nucleus, $\varphi^{(\pm)}$, which is caused by the Coulomb interaction. The operator

$$\begin{aligned} \mathcal{T}_{\text{nuc}} &= V_{\text{nuc}} + V_{\text{nuc}}(E_a - H_0 - V_{\text{Coul}} + i\varepsilon)^{-1}\mathcal{T}_{\text{nuc}} \\ &= V_{\text{nuc}} + V_{\text{nuc}}(E_a - H_0 - V_{\text{Coul}} - V_{\text{nuc}} + i\varepsilon)^{-1}V_{\text{nuc}}, \end{aligned} \quad (64)$$

and the wave functions $\varphi_a^{(\pm)}$ describe the interaction between particles and atoms via the Coulomb interaction alone ($V_{\text{nuc}} = 0$) [6, 15, 82]:

$$\varphi_a^{(\pm)} = \Phi_a + (E_a - H_0 \pm i\varepsilon)^{-1}V_{\text{Coul}}\varphi_a^{(\pm)}, \quad (65)$$

the wave function $\varphi_a^{(+)}$ at large distances has the asymptotics of a diverging spherical wave, and the wave function $\varphi_a^{(-)}$ at large distances has the asymptotics of a converging spherical wave [6, 15, 82].

Let us give a more detailed consideration of the matrix element $\langle \varphi_b^{(-)} | \mathcal{T}_{\text{nuc}} | \varphi_a^{(+)} \rangle$. Since nuclear forces are short-range, the radius of the domain of integration in this matrix element is of the order of the nuclear radius (of the order of the radius of action of nuclear forces in the case of the proton). The Coulomb interaction, V_{Coul} , in this domain is noticeably smaller than the energy of nuclear interaction, V_{nuc} . We can therefore neglect the Coulomb energy in the first approximation in the denominator of Eq. (64), as compared to V_{nuc} .

As a result, the operator \mathcal{T}_{nuc} is reduced to the operator describing a purely nuclear interaction between the incident particle and the nucleus. The effect of Coulomb forces on nuclear interaction is described by wave functions $\varphi_{ba}^{(\pm)}$ (distorted-wave approximation [82]).

In the range of antiproton energies of hundreds of kiloelectronvolts and less, the de Broglie wavelength for antiprotons is larger than the nuclear radius. Therefore, in Eq. (62) for $\mathcal{T}_{ba}^{\text{nuc}}$, one can remove the wave functions $\varphi_{a(b)}^{(\pm)}$ outside the sign of integration over the coordinate of the antiproton center of mass, \vec{R}_p , at the location point of the nuclear center of mass, \vec{R}_{nuc} . As a result, one may write the following relationship:

$$\mathcal{T}_{ba}^{\text{nuc Coul}} = g_{ba}\mathcal{T}_{ba}^{\text{nuc}} = \langle \varphi_b^{(-)}(\vec{R}_p = \vec{R}_{\text{nuc}}) | \varphi_a^{(+)}(\vec{R}_p = \vec{R}_{\text{nuc}}) \rangle \mathcal{T}_{ba}^{\text{nuc}}, \quad (66)$$

where $\mathcal{T}_{ba}^{\text{nuc}}$ is the matrix element describing a purely nuclear interaction (in the absence of Coulomb interaction) between the incident particle and the nucleus. The factor $g_{ba} = \langle \varphi_b^{(-)}(\vec{R}_p = \vec{R}_{\text{nuc}}) | \varphi_a^{(+)}(\vec{R}_p = \vec{R}_{\text{nuc}}) \rangle$ appearing in (66) defines the probability to find the antiproton (the negative hyperon, e.g. Ω^- , Σ^-) at the location of the nucleus.

From (62)–(66) one can derive the following expression for the amplitude of coherent elastic zero-angle scattering:

$$f(0) = -\frac{m}{2\pi\hbar^2} g_{aa} \mathcal{T}_{aa}^{\text{nuc}} = g_{aa} f_{\text{nuc}}(0), \quad (67)$$

where $f_{\text{nuc}}(0)$ is the amplitude of particle scattering by the nucleus in the absence of Coulomb interaction, and $g_{aa} = \langle \varphi_b^{(-)}(\vec{R}_p = \vec{R}_{\text{nuc}}) | \varphi_a^{(+)}(\vec{R}_p = \vec{R}_{\text{nuc}}) \rangle$ is the probability to find the particle incident onto the nucleus at the location of the nucleus.

Thus, the Coulomb interaction leads to a change in the value of the amplitude of nuclear forward scattering. Let us estimate the magnitude of this change.

According to [6], when a particle moves in the Coulomb field, the probability g_{aa} can be written in the form:

- for the case of repulsion, i.e., scattering of similarly charged particles

$$g_{aa}^{\text{rep}} = \frac{2\pi}{\kappa(e^{\frac{2\pi}{\kappa}} - 1)}, \quad \kappa = \frac{v}{Z\alpha c}, \quad (68)$$

where v is the particle velocity, Z is the charge of the nucleus, α is the fine structure constant, c is the speed of light;

- for the case of attraction

$$g_{aa}^{\text{att}} = \frac{2\pi}{\kappa(1 - e^{-\frac{2\pi}{\kappa}})}. \quad (69)$$

With decreasing particle energy (velocity), κ diminishes, and for such values of κ when $\frac{2\pi}{\kappa} \geq 1$, one can write

$$g_{aa}^{\text{rep}} = \frac{2\pi\alpha Zc}{v} e^{-\frac{2\pi\alpha Zc}{v}}, \quad g_{aa}^{\text{att}} = \frac{2\pi\alpha Zc}{v}. \quad (70)$$

According to Eq. (104), with decreasing energy of positively charged particles, the amplitude $f(0)$ diminishes rapidly because of repulsion. For negatively charged particles, the amplitude grows with decreasing particle energy (velocity).

These results for the amplitude $f(0)$ generalize a similar well-known relationship for taking account of the Coulomb interaction effect on the cross section of inelastic processes, σ_r , [6].

Thus, in the range of low energies, the amplitude $f(0)$ of antiproton (negative hyperon) scattering by the nucleus can be presented in the form:

$$f(0) = \frac{2\pi\alpha Zc}{v} f_{\text{nuc}} = \frac{2\pi\alpha Zc}{v} \text{Re} f_{\text{nuc}} + \frac{i}{2} \frac{\alpha Z}{\lambda_c} \sigma_{\text{tot}}, \quad (71)$$

where $\lambda_c = \frac{\hbar}{mc}$, σ_{tot} is the total cross section of nuclear interaction between the particle and the scatterer. In deriving Eq. (105), the optical theorem $\text{Im} f(0) = \frac{k}{4\pi} \sigma_{\text{tot}}$ was applied, where k is the wave number of the incident particle.

4.4. Effective Potential Energy of Negatively Charged Particles in Matter

Using the amplitude $f(0)$, one can write the expression for the refractive index \hat{n} of a spin particle in matter with polarized nuclei (see (1)), as well as the expression for the effective potential energy \hat{U}_{eff} of interaction between this particle and matter [16, 37, 73–75]:

$$\hat{n}^2 = 1 + \frac{4\pi N}{k^2} \hat{f}(0) \quad \text{and} \quad \hat{U}_{\text{eff}} = -\frac{2\pi\hbar^2}{m} N \hat{f}(0), \quad (72)$$

where $\hat{f}(0) = \frac{2\pi\alpha Zc}{v} \hat{f}_{\text{nuc}}(0)$ is the amplitude of coherent elastic forward scattering, which is the operator in the particle spin space.

The amplitude $\hat{f}(0)$ depends on the vector polarization \vec{P}_t of the target nuclei and can be presented in the form:

$$\hat{f}(0) = A_0 + A_1(\hat{S}\vec{P}_t) + A_2(\hat{S}\vec{e})(\vec{e}\vec{P}_t), \quad (73)$$

where A_0 is the scattering amplitude independent of the incident particle spin, \hat{S} is the particle spin operator, and \vec{e} is the unit vector in the direction of the particle momentum. If the spin of the target nuclei $I \geq 1$, then the addition depending on the target tensor polarization also appears [37, 73].

Correspondingly, the effective potential energy \hat{U}_{eff} of particle interaction with polarized matter looks like

$$\hat{U}_{\text{eff}} = -\frac{2\pi\hbar^2}{m}N(A_0 + A_1(\hat{S}\vec{P}_t) + A_2(\hat{S}\vec{e})(\vec{e}\vec{P}_t)). \quad (74)$$

Expression (74) can be written as

$$\hat{U}_{\text{eff}} = U_{\text{eff}} + \hat{V}_{\text{eff}}(\vec{P}_t), \quad (75)$$

where

$$U_{\text{eff}} = -\frac{2\pi\hbar^2}{m}NA_0, \quad \hat{V}_{\text{eff}}(\vec{P}_t) = -\vec{\mu}\vec{G} = -\frac{\mu}{S}(\hat{S}\vec{G}), \quad (76)$$

with μ being the particle magnetic moment,

$$\vec{G} = \frac{2\pi\hbar^2S}{m\mu}N(A_1\vec{P}_t + A_2\vec{e}(\vec{e}\vec{P}_t)). \quad (77)$$

Recall now that the energy of interaction between the magnetic moment μ and the magnetic field \vec{B} is as follows:

$$V_{\text{mag}} = -(\vec{\mu}\vec{B}) = -\frac{\mu}{S}(\hat{S}\vec{B}). \quad (78)$$

Expressions (76) and (77) are identical. Therefore, \vec{G} can be interpreted as the effective pseudomagnetic field acting on the spin of the particle moving in matter with polarized nuclei and appears due to nuclear interaction between the incident particles and the scatterers. Similarly to particle spin precession in an external magnetic field, particle spin precesses in field \vec{G} . This phenomenon was called the nuclear precession of the particle spin, first described for slow neutrons in [16] and then observed in [17, 18].

From Eq. (105) it follows that in the range of low energies, \hat{U}_{eff} can be presented in the form:

$$\hat{U}_{\text{eff}} = \frac{2\pi\alpha Zc}{v}\hat{U}_{\text{eff}}^{\text{nuc}}, \quad (79)$$

where $\hat{U}_{\text{eff}}^{\text{nuc}}$ has the same form as that in Eq. (74) with the amplitudes A_0 , A_1 , and A_2 replaced by A_0^{nuc} , A_1^{nuc} and A_2^{nuc} calculated ignoring the Coulomb interaction.

According to Eq. (79), with decreasing particle velocity, \hat{U}_{eff} grows as well as the field \vec{G} , and particle spin precession in this field is $\Omega_{\text{pr}} \sim \text{Re } G \sim \frac{1}{v}$.

Let us take a somewhat different view of this issue. According to Eqs. (74), (76), and (79), U_{eff} depends on the orientation of vectors \vec{e} and \vec{P}_t . As follows from Eq. (74), two simpler cases

can be distinguished: $\vec{e} \perp \vec{P}_t$ and $\vec{e} \parallel \vec{P}_t$. Accordingly, we have two cases for the effective interaction energy: $\hat{U}_{\text{eff}}^\perp$ and $\hat{U}_{\text{eff}}^\parallel$:

$$\hat{U}_{\text{eff}}^\perp = -\frac{2\pi\hbar^2}{m} N(A_0 + A_1(\hat{S}\vec{P}_t)), \quad (80)$$

$$\hat{U}_{\text{eff}}^\parallel = -\frac{2\pi\hbar^2}{m} N(A_0 + (A_1 + A_2)(\hat{S}\vec{P}_t)), \quad (81)$$

Let us choose the quantization axis to be parallel to the polarization vector \vec{P}_t . Hence, for particle states with magnetic quantum number M_s , one can write the below expressions for $U_{\text{eff}}^\perp(M_s)$ and $U_{\text{eff}}^\parallel(M_s)$, which follow from Eq. (80):

$$U_{\text{eff}}^\perp(M_s) = -\frac{2\pi\hbar^2}{m} N(A_0 + A_1 M_s P_t), \quad (82)$$

$$U_{\text{eff}}^\parallel(M_s) = -\frac{2\pi\hbar^2}{m} N(A_0 + (A_1 + A_2) M_s P_t). \quad (83)$$

In the case of antiprotons ($M_s = \pm\frac{1}{2}$), from Eqs. (82), (83) one can obtain two values of the effective potential energy, depending on the antiproton spin orientation. The difference of the real parts of these energies defines the spin precession frequency of the antiproton in matter with polarized nuclei:

$$\Omega_{\text{pr}}^\perp = \frac{\text{Re}(U_{\text{eff}}^\perp(+\frac{1}{2}) - U_{\text{eff}}^\perp(-\frac{1}{2}))}{\hbar} = -\frac{2\pi\hbar}{m} \frac{2\pi\alpha Zc}{v} N P_t \text{Re} A_{1\text{nuc}}, \quad (84)$$

$$\Omega_{\text{pr}}^\parallel = \frac{\text{Re}(U_{\text{eff}}^\parallel(+\frac{1}{2}) - U_{\text{eff}}^\parallel(-\frac{1}{2}))}{\hbar} = -\frac{2\pi\hbar}{m} \frac{2\pi\alpha Zc}{v} N P_t \text{Re}(A_{1\text{nuc}} + A_{2\text{nuc}}). \quad (85)$$

4.5. Spin Rotation Angle of Low-Energy Antiprotons in Polarized Matter

Let us estimate the magnitude of the effect. Since in the range of low energies, the de Broglie wavelength of a particle is much larger than the nuclear radius, in making estimations we shall concentrate on S -scattering (one should bear in mind, though, that the analysis of the antiproton–proton interaction in protonium has shown that at low energies, P -waves also contribute to antiproton–proton interaction [70]). In S -wave scattering, the amplitude A_2^{nuc} equals zero.

The amplitudes A_0^{nuc} and A_1^{nuc} can be expressed in terms of the amplitudes a^+ and a^- , where a^+ is the scattering amplitude in the state with total momentum $I + \frac{1}{2}$, and a^- is the same in the state with total momentum $I - \frac{1}{2}$ (I is the nuclear spin) [6]:

$$A_0^{\text{nuc}} = \frac{I+1}{2I+1} a^+ + \frac{I}{2I+1} a^-, \quad A_1^{\text{nuc}} = \frac{2I}{2I+1} (a^+ - a^-). \quad (86)$$

As a consequence, one can write the following expression for Ω_{pr} :

$$\Omega_{\text{pr}} = -\frac{2\pi\hbar}{m} \frac{2\pi\alpha Zc}{v} N P_t \frac{2I}{2I+1} \text{Re}(a^+ - a^-). \quad (87)$$

When antiprotons pass through a hydrogen target ($I = \frac{1}{2}$), we have:

$$\Omega_{\text{pr}} = \frac{\pi\hbar}{m} \frac{2\pi\alpha c}{v} N P_t \text{Re}(a^+ - a^-). \quad (88)$$

The factor $\frac{2\pi\alpha c}{v}$ makes (88) different from the equation for spin precession frequency of slow neutrons moving in a target with polarized protons.

Recall that in the range of low energies, the amplitudes a^+ and a^- are often expressed in terms of the scattering lengths b^+ and b^- [6]:

$$a^+ = -b^+ \quad \text{and} \quad a^- = -b^-.$$

When the neutron is scattered by the proton, $b^+ = 5.39 \cdot 10^{-13}$ cm, $b^- = -2.37 \cdot 10^{-12}$ cm [6]. As a result, in the case of $n-p$ scattering, for the amplitude A_0^{nuc} (see Eq. (86)) we have $A_0^{\text{nuc}} \approx -1.9 \cdot 10^{-13}$ cm, while for the amplitude A_1^{nuc} , we have $A_1^{\text{nuc}} \approx 1.46 \cdot 10^{-12}$ cm. As can be seen, $A_1 \gg |A_0|$.

In the case of antiproton-proton interaction, the spin-independent part of the scattering length, “ b ”, is $b \sim 10^{-13}$ cm [70], which is comparable with $|A_0^{\text{nuc}}|$ for the case of $n-p$ scattering. For antiproton scattering by the proton, the magnitude of A_1 is unknown in the considered range of low energies.

Let us assume for further estimations that $\text{Re } A_1^{\text{nuc}}$ in the case of $\bar{p}-p$ scattering is comparable with $\text{Re } A_1^{\text{nuc}}$ in the case of $n-p$ scattering, i.e., $\text{Re } A_1^{\text{nuc}}$ is of the order of 10^{-12} cm.

We can finally obtain the following estimate for the antiproton spin precession frequency in matter with polarized protons:

$$\Omega_{\text{pr}} = \frac{2\pi\alpha c}{v} \frac{2\pi\hbar}{m} N P_t \text{Re } A_1 \simeq \frac{2\pi\alpha c}{v} 6 \cdot 10^7 \frac{N}{N_l} P_t, \quad (89)$$

where N_l is the number of atoms in 1 cm³ of liquid hydrogen, $N_l \simeq 4.25 \cdot 10^{22}$. It will be recalled that $\frac{2\pi\alpha c}{v} \gg 1$ for slow antiprotons with velocity $v < 10^9$ cm/s.

Let us estimate now the spin rotation angle of the antiproton. An antiproton moving in Hydrogen may be captured by a proton and forms a bound state, protonium. We shall therefore estimate the magnitude of the spin rotation angle during the characteristic time τ necessary for the antiproton to be captured into a bound state: $\tau \sim \frac{1}{N v \sigma_{\text{pr}}}$, where σ_{pr} is the cross section for protonium formation.

The antiproton spin rotation angle ϑ during this time can be estimated using the formula:

$$\vartheta \sim \Omega_{\text{pr}} \tau = \frac{2\pi\hbar}{m} \frac{2\pi\alpha c}{v^2} P_t \frac{\text{Re } A_1}{\sigma_{\text{pr}}} = \frac{2\pi^2\hbar}{E} \alpha c \frac{\text{Re } A_1}{\sigma_{\text{pr}}} P_t, \quad (90)$$

where $E = \frac{mv^2}{2}$ is the antiproton energy.

Presently, antiproton beams with the energy of hundreds of electronvolts and lower [70, 71] are available. According to [83], in the range of energies higher than 10 eV, the cross section of antiproton capture by hydrogen with the formation of protonium is $\sigma_{\text{pr}} \leq 10^{-18}$ cm². As a result, the spin rotation angle for antiprotons with an energy of 10 eV can be estimated as $\vartheta \simeq 6 \cdot 10^{-2} P_t$. For antiproton energies of 20 eV, the same is $\vartheta \sim 3 \cdot 10^{-1} P_t$. When antiprotons are decelerated in a polarized gaseous target, the degree of proton polarization is close to unity. As a consequence, the rotation angle ϑ in such a target reaches quite appreciable values $\vartheta \sim 10^{-1}$, giving hope for experimental observation of the effect.

Thus, this effect will be applicable for $\text{Re } A_1$ measurement in the region where the scattering experiments are practically impossible to perform. One of the possibilities to study the spin rotation effect and to analyze the polarization state of antiprotons (hyperons) under deceleration in matter consists in the use of polarized targets with nuclei having a large Z (in fact, a thin film, in which case nuclei in a static magnetic field can be polarized far more readily) and in

investigation of the intensity of the annihilation product formation (angular distribution) as a function of the spin direction of the incident particle relative to the spin of the target nucleus.

Let us note in conclusion that the deceleration of antiprotons in Hydrogen finally results in the formation of a neutral atom — protonium [70]. Protonium undergoes interactions with a nuclear pseudomagnetic field of polarized matter in a similar way as a neutron. This interaction leads to splitting and shifting of energy levels of excited and ground states of protonium (and similar atoms, such as $\bar{p}d$, $\bar{p}^3\text{He}$), as well as to spin rotation and oscillations of the excited and ground states of these atoms.

Thus, the above analysis shows that since the real part of the coherent elastic forward scattering amplitude increases under deceleration of \bar{p} (hyperon), the effective potential energy of interaction between the particle and matter grows, along with the pseudomagnetic field. As a result, the spin rotation angle ϑ in the nuclear pseudomagnetic field of matter reaches the value $\vartheta \sim 10^{-1}$, which gives hope to experimentally observe the real part of the amplitude in the range where the scattering experiments are practically impossible to perform.

4.6. Mirror Reflection of Antiprotons from the Surface of Matter

Before we start to consider mirror reflection of antiprotons from the surface formed by the vacuum-matter boundary, let us make some general remarks about mirror reflection of particles.

It is well known [27, 28] that the mirror-reflection coefficient R is defined by the refractive index n of a wave in matter. For such particles as, say, neutrons (for light and γ -quanta, whose polarization is orthogonal to the mirror-reflection plane), the mirror-reflection coefficient has the form [27, 28]

$$R = |F|^2, \quad (91)$$

$$F = -\frac{\sqrt{n^2 - \cos^2 \varphi} - \sin \varphi}{\sqrt{n^2 - \cos^2 \varphi} + \sin \varphi}, \quad (92)$$

where φ is the grazing angle (see Fig. 7).

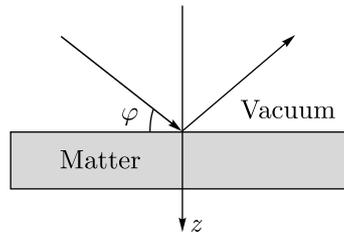


Figure 7. Mirror reflection at the vacuum-matter boundary.

The refractive index of particles or γ -quanta can be expressed as follows [15, 82] (see also (1)):

$$n^2 = 1 + \frac{4\pi\rho}{k^2} f(0),$$

where ρ is the number of scatterers (nuclei, atoms) per cubic centimeter of matter and $f(0)$ is the amplitude of zero-angle coherent elastic scattering. According to (91)–(92), the mirror-reflection coefficient is determined by the zero-angle scattering amplitude. But at the same time, the wave scattered in the mirror-reflection direction obviously makes a certain angle ϑ relative to the incident direction. Thus, the amplitude of this wave is determined by the superposition of waves scattered at the angle ϑ , i.e., instead of $f(0)$, the amplitude F is now determined by the amplitude $f(\vartheta)$ of coherent elastic scattering at the angle ϑ .

Indeed, let us consider elastic scattering of a wave $e^{i\vec{k}\vec{r}} = e^{i\vec{k}_\perp \vec{r}_\perp} e^{ik_z z}$ by a set of scatterers located in the plane $z = z_0$. Here \vec{k}_\perp is the wave vector's component perpendicular to the z -axis (parallel to the plane where the scatterers are placed), k_z is the component of the particle's wave vector that is parallel to the z -axis. Upon summation of the spherical waves produced by the scatterers in the plane $z = z_0$, we obtain the following expression for the amplitude of a mirror-reflected wave F_1 [37]:

$$F_1 = \frac{2\pi i \rho'}{k_z} f(\vec{k}' - \vec{k}) e^{2ik_z z_0}, \quad (93)$$

where $|\vec{k}'| = |\vec{k}|$, \vec{k}' is the wave vector of the scattered particle, which has the components $\vec{k}'_\perp = \vec{k}'_\perp$ and $k'_z = -k_z$, and ρ' is the density of scatterers in the considered plane (the number of scatterers in cm^2 of the plane).

If the layer $[0, z]$ contains m number of planes, then the amplitude of the wave reflected by these planes can be written in the form

$$F = \frac{2\pi i \rho'}{k_z} f(\vec{k}' - \vec{k}) \sum_m e^{2ik_z z_m}, \quad (94)$$

Passing to a continuous distribution of planes in the layer $[0, z]$, i.e., replacing the summation in (94) by integration, we finally obtain [37]

$$F = -\frac{\pi \rho}{k_z^2} f(\vec{k}' - \vec{k}) = -\frac{4\pi \rho}{|\vec{k}' - \vec{k}|^2} f(\vec{k}' - \vec{k}). \quad (95)$$

According to (95), microscopic summation of waves scattered at the vacuum-matter boundary yields the formation of a wave reflected in the direction determined by the laws of classical optics. Against (91), its amplitude is defined not by the amplitude $f(0)$, but by the amplitude of scattering at a nonzero angle ϑ equal to a double grazing angle φ , i.e., $\vartheta = 2\varphi$ [37]. Equation (91) is valid for thermal and ultracold neutrons because of the prevailing isotropic S-scattering by nuclei ($f(\vartheta) = f(0)$). It is also valid for photons, because at wavelengths much greater than the size of the atom, dipole scattering, which is also isotropic under the considered polarization, occurs.

According to (94), the amplitude F of mirror reflection rises with decreasing $k_z = k \sin \varphi$, i.e., with decreasing grazing angle φ . When F reaches the values close to unity, equation (95) for F is no longer valid. Mirror reflection for this range of angles is considered, e.g., in [37].

In view of (95), the amplitude F is close to unity in the range where the grazing angles φ satisfy the condition

$$\sin^2 \varphi \sim \frac{\pi \rho |f|}{k^2}. \quad (96)$$

If in estimating the angle φ we use a typical value of the amplitude of scattering via nuclear interaction, $|f| \sim 10^{-12}$ cm, then (96) readily yields that for antiproton energies from 100 to 1000 eV, the value of F can be close to unity only when the grazing angles are very small: $\varphi \leq 10^{-4} \div 10^{-5}$. Further in our consideration we shall demonstrate that owing to the interference of Coulomb and nuclear interactions and the increase in the amplitude of nuclear scattering of antiprotons with decreasing energy [75], the coefficient of mirror reflection for antiprotons becomes noticeable even at much larger grazing angles φ , making it possible to use the phenomenon of mirror reflection to investigate scattering of slow antiprotons by nuclei [84].

According to (95), the mirror-reflection coefficient R can be written as follows:

$$R = |F|^2 = \left| \frac{\pi\rho}{k_z^2} f(\vec{k}' - \vec{k}) \right|^2 = \frac{\pi^2 \rho^2}{k^4 \sin^4 \varphi} |f(\vec{k}' - \vec{k})|^2. \quad (97)$$

There are two interactions responsible for antiproton scattering by nuclei (atoms): Coulomb and nuclear (here we neglect the magnetic interaction of antiproton and electron spins). Consequently, the scattering amplitude f can be presented as a sum of two amplitudes:

$$f = f_{\text{Coul}} + f_N, \quad (98)$$

where f_{Coul} is the amplitude of a purely Coulomb scattering and f_N is the amplitude related to nuclear interaction (it contains the contribution from Coulomb interaction that affects nuclear scattering [75]). Let us note that the contribution to the formation of a mirror-reflected wave comes from elastic scattering, in which the state of the target does not change. As is known, the scattering amplitude in this case can be presented as a product of the amplitude of elastic scattering by an infinite-mass nucleus (atom) (the reduced mass equals the mass of the incident particle) into the Debye–Waller factor $e^{-w(\vec{k}' - \vec{k})}$ describing the effect produced by thermal oscillations of nuclei (atoms) in matter on the process of scattering [27]. In subsequent consideration, by the amplitude f we shall mean scattering by an infinite-mass nucleus. As a result, we have ($\varphi \ll 1$)

$$R = \frac{\pi^2 \rho^2}{k^4 \varphi^4} \left(|f_{\text{Coul}}(\vec{k}' - \vec{k})|^2 + 2\text{Re} f_{\text{Coul}}(\vec{k}' - \vec{k}) f_N^* + |f_N|^2 \right) e^{-2w(\vec{k}' - \vec{k})}. \quad (99)$$

For further consideration we need to compare the amplitudes of Coulomb and nuclear elastic scattering of antiprotons by nuclei (atoms).

4.7. The Amplitude of Antiproton Scattering by a Nucleus (Atom) at Low Energy

The scattering amplitude relates to the \mathcal{T} -matrix as [15, 82]

$$f_{ba} = -\frac{m}{2\pi\hbar^2} \langle \Phi_b | \mathcal{T} | \Phi_a \rangle, \quad (100)$$

where $|\Phi_{a(b)}\rangle$ is the wave function describing the initial (final) state of the system “incident particle–atom (nucleus)”. The wave functions $|\Phi_a\rangle$ are the eigenfunctions of the Hamiltonian $H_0 = H_p(\vec{r}_p) + H_A(\vec{\xi}, \vec{r}_{\text{nuc}})$, i.e., $H_0|\Phi_a\rangle = E_a|\Phi_a\rangle$; $H_p(\vec{r}_p)$ is the Hamiltonian of the particle incident onto the target; \vec{r}_p is the particle coordinate; $H_A(\vec{\xi}, \vec{r}_{\text{nuc}})$ is the atomic (nuclear) Hamiltonian; $\vec{\xi}$ is the set of coordinates of the atomic electron; \vec{r}_{nuc} is the set of coordinates describing the atomic nuclei.

The Hamiltonian H describing the particle–nucleus interaction reads as (61).

Using the same approach as that applied for deriving (62)–(66), one can obtain the following expression for the scattering amplitude [84]:

$$f_{ba}^N = -\frac{m}{2\pi\hbar} g_{ba} \mathcal{T}_{ba}^{\text{nuc}} = g_{ba} f_{ba}^{\text{nuc}}, \quad (101)$$

where f_{ba}^{nuc} is the amplitude of particle scattering by the nucleus in the absence of Coulomb interaction.

Thus, Coulomb interaction leads to a change in the value of the amplitude of antiproton-nucleus scattering. Let us estimate the magnitude of this change.

In what follows, we shall be primarily concerned with elastic scattering. In this case $|\vec{k}| = |\vec{k}'|$, and it follows from the expression given in [6] for the wave functions $\varphi_b^{(-)}$ and $\varphi_a^{(+)}$, which describe particle scattering in the Coulomb field, that we can derive the following relationships for g_{ba} :

- for the case of repulsion, i.e., elastic scattering of similarly charged particles

$$g_{ba}^{\text{rep}} = \frac{2\pi}{\kappa(e^{\frac{2\pi}{\kappa}} - 1)}, \quad \kappa = \frac{v}{Z\alpha c}, \quad (102)$$

where v is the particle velocity, Z is the charge of the nucleus, α is the fine structure constant, c is the speed of light;

- for the case of attraction

$$g_{ba}^{\text{att}} = \frac{2\pi}{\kappa(1 - e^{-\frac{2\pi}{\kappa}})}. \quad (103)$$

With decreasing particle energy (velocity), κ diminishes, and for such values of κ when $\frac{2\pi}{\kappa} \geq 1$, one can write

$$g_{ba}^{\text{rep}} = \frac{2\pi\alpha Zc}{v} e^{-\frac{2\pi\alpha Zc}{v}}, \quad g_{ba}^{\text{att}} = \frac{2\pi\alpha Zc}{v}. \quad (104)$$

According to (101), with decreasing energy of positively charged particles, the amplitude f_{ba}^N diminishes rapidly because of repulsion. For negatively charged particles, the amplitude grows with decreasing particle energy (velocity).

These results for the amplitude f_{ba}^N generalize a similar well-known relationship for taking account of the Coulomb interaction effect on the cross section of inelastic processes, σ_r , [6].

Thus, in the range of low energies, the amplitude $f(\vec{k}' - \vec{k})$ of antiproton (negative hyperon) scattering by a nucleus can be presented in the form (for antiproton scattering in ferromagnets, the magnetic interaction of antiprotons with electrons in atoms should also be considered):

$$f(\vec{k}' - \vec{k}) = f_{\text{Coul}}(\vec{k}' - \vec{k}) + f_N(\vec{k}' - \vec{k}), \quad (105)$$

where

$$f_N(\vec{k}' - \vec{k}) = \frac{2\pi\alpha Zc}{v} f_{\text{nuc}}(\vec{k}' - \vec{k}). \quad (106)$$

Using (106) and the expression for f_{Coul} in [6], we can obtain the following expression for the modulus of the Coulomb (Rutherford) scattering amplitude in the range of small scattering angles ($\vartheta \ll 1$, $\vartheta > \frac{1}{kR}$):

$$|f_{\text{Coul}}(\vec{k}' - \vec{k})| = \frac{2Ze^2}{mv^2\vartheta^2} = \frac{Ze^2}{2mv^2\varphi^2} \quad (107)$$

and then write the ratio for these amplitudes as

$$\frac{|f_N|}{|f_{\text{Coul}}|} = 4\pi k\varphi^2 |f_{\text{nuc}}|. \quad (108)$$

As a result, using the estimate $|f_{\text{nuc}}| \approx 10^{-12}$ cm for the characteristic nuclear amplitude, $k \leq 10^{10} \div 10^{11}$, $\varphi \sim 10^{-1}$, we can estimate the ratio $\frac{|f_{\text{nuc}}|}{|f_{\text{Coul}}|}$ as $\frac{|f_{\text{nuc}}|}{|f_{\text{Coul}}|} \leq 3 \cdot 10^{-4} \div 10^{-2}$.

In view of the above estimate, we can recast the expression for the coefficient of reflection as follows:

$$R = R_{\text{Coul}} \left(1 + 2 \frac{\text{Re} e^{i\beta} f_{\text{nuc}}^*}{|f_{\text{Coul}}|} \right) e^{-2w(\vec{k}' - \vec{k})}, \quad (109)$$

where $R_{\text{Coul}} = \frac{\pi^2 \rho^2}{k^4 \varphi^4} \left| f_{\text{Coul}}(\vec{k}' - \vec{k}) \right|^2$ is the coefficient of mirror reflection due to a purely Coulomb interaction, $f_{\text{Coul}} = |f_{\text{Coul}}| e^{i\beta}$, and the term in (99) that is proportional to $\frac{|f_{\text{nuc}}|^2}{|f_{\text{Coul}}|}$ is dropped for its smallness. Let us recall that (109) holds true for such values of R_{Coul} that are much less than unity.

Thus, the coefficient of mirror reflection contains two contributions: one comes from purely Coulomb interaction, and the other is due to the Coulomb-nuclear interference. This makes it possible to obtain data about the amplitude of nuclear scattering of antiprotons by the nucleus from experiments investigating the angular and energy dependence of R on the grazing angle and the energy of antiprotons.

4.8. Spin Polarization of Antiprotons Reflected from the Vacuum-Matter Boundary

As is well known, the spin-orbit interaction during scattering causes the initially nonpolarized particle beam to become polarized [6]. In this case, the polarization vector of particles appears to be orthogonal to the scattering plane, i.e., the plane containing vectors \vec{k} and \vec{k}' . If the particle beam had a nonzero polarization vector before the interaction, then a left-right asymmetry in the intensity of particle scattering is observed. For slow neutrons, the spin-orbit interaction is caused by the interaction V_{so} of the neutron magnetic moment and the nuclear electric field [62]

$$V_{so}^{\text{neut}} = i \frac{\mu_n \hbar}{mc} \vec{\sigma} [\vec{E}(\vec{r}) \vec{\nabla}_{\vec{r}}], \quad (110)$$

where μ_n is the neutron magnetic moment, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices, \vec{E} is the electric field at point \vec{r} where the neutron is located, and m is the neutron's mass. First experiments to observe the effect of spin-orbit interaction on neutron scattering were performed by C.G. Shull [85]. (For further experiments see, e.g., [37]).

The presence of charge in antiprotons appreciably affects the dependence of spin-orbit interaction on their magnetic moment. The energy of spin-orbit interaction of antiprotons with nuclei is determined by the electric field and has the form [62]

$$V_{so}^{ap} = - \left(\mu' + \frac{e\hbar}{4mc} \right) \left(\vec{\sigma} \left[\vec{E} \frac{\hat{p}}{m} \right] \right), \quad (111)$$

where μ' is the anomalous part of the antiproton's magnetic moment, $e = -|e|$ is the antiproton charge, $|e|$ is the electron charge, and $\hat{p} = -i\hbar \vec{\nabla}$ is the momentum operator of the antiproton.

The energy dependence of the amplitude of spin-orbit scattering also changes noticeably through the interference of Coulomb and spin-orbit interactions. The energy dependence of the contribution coming from the antiproton-nucleus strong interaction to the amplitude of spin-orbit scattering changes as well.

The expression describing the amplitude of spin-orbit scattering in a general form reads:

$$F_{so} = F_{so} \vec{\sigma} [\vec{k}' \times \vec{k}] = F_{so} \vec{\sigma} [\vec{q} \times \vec{k}], \quad (112)$$

where $\vec{q} = \vec{k}' - \vec{k}$ is the momentum transfer. It should be noted that the general form of (112) is clear even without calculations and follows from the symmetry considerations, being valid for all spin particles.

Thus, the amplitude of mirror reflection from matter with nonpolarized nuclei is a sum of three amplitudes: the amplitude of Coulomb scattering (or the amplitude of magnetic scattering by electrons in ferromagnets), the particle-spin-independent amplitude of nuclear scattering, and the amplitude of spin-orbit scattering. Hence, the coefficient of mirror reflection contains the contributions coming from these amplitudes and their interference. Let us consider the contribution coming to the coefficient of mirror reflection from the interference of Coulomb and spin-orbit interactions. Obviously, it is proportional to $\vec{\sigma}[\vec{k}' \times \vec{k}]$.

Let us choose the quantization axis to be parallel to the reflecting plane (the plane containing vectors \vec{k}' and \vec{k}). It follows from (99) and (112) that due to Coulomb-nuclear interference, the reflection coefficients $R_{\uparrow\uparrow}$ and $R_{\downarrow\uparrow}$ for antiprotons with spins parallel and antiparallel to the direction of the axial vector $[\vec{k} \times \vec{k}']$ will differ. Let a nonpolarized antiproton beam be incident on the target. Such a beam is represented by a coherent sum of two beams with spins parallel and antiparallel to $[\vec{k} \times \vec{k}']$. A mirror-reflected beam appears partially polarized, and the degree to which the beam is polarized is determined by the difference $R_{\uparrow\uparrow} - R_{\downarrow\uparrow}$ of mirror-reflection coefficients:

$$p = \frac{R_{\uparrow\uparrow} - R_{\downarrow\uparrow}}{R_{\uparrow\uparrow} + R_{\downarrow\uparrow}}. \tag{113}$$

When estimating the magnitude of the effect, we should take into account that in the Born approximation, the amplitude F_{so} is purely imaginary, while the amplitude F_{Coul} is real, and so it is important that the imaginary part of F_{Coul} be considered. It can be readily found, since we know that in the range of small scattering angles, $\text{Im}F_{Coul}$ can be set equal to $\text{Im}F_{Coul}(0)$ ($\text{Im}F_{Coul}(0)$ is the amplitude of Coulomb scattering by a nucleus (atom) at zero angle).

According to the optical theorem, $\text{Im}F_{Coul}(0) = \frac{k}{4\pi}\sigma_{tot}$. The total cross section of scattering by a screened Coulomb potential, $V_{Coul} = \frac{Ze^2}{r}e^{-\frac{r}{R_A}}$, can be written as

$$\sigma = 16\pi \left(\frac{mZe^2R_A^2}{\hbar^2} \right)^2 \frac{1}{1 + \frac{8mER_A^2}{\hbar^2}}, \tag{114}$$

where E is the particle energy.

It follows from (114) that for antiproton energies greater than the characteristic energy $E_A = \frac{\hbar^2}{4zmR_A^2} \approx 10^{-2}$ eV, the scattering cross section $\sigma \sim \frac{1}{E} \sim \frac{1}{v}$. As a result, $\text{Im}F_{Coul} \simeq \text{const}$. The amplitude of spin-orbit interaction contains the term $\frac{2\pi\alpha Zc}{v}$ [see (101), (106)] that increases the amplitude of spin-orbit scattering of antiprotons by nuclei (atoms), making it grow as $\sim \frac{1}{v}$ in the range of small grazing angles (the momentum transfer in this case is required to be $q > \frac{1}{R_A}$). As a result, the contribution of spin-orbit scattering to the coefficient of mirror reflection can be estimated as [84]:

$$R_{so} \sim \frac{\pi\rho p^2}{k^4\varphi^4} |\text{Im}f_{Coul}|^2 \frac{f_{so}}{\text{Im}f_{Coul}} \approx 10^{-1} \div 10^{-2}.$$

Consequently, this process can be used to obtain polarized antiprotons in this low energy range.

Let us note here that when a polarized antiproton beam falls on the surface, the particles' polarization vector rotates about the quantization axis, i.e., about the direction $[\vec{k}' \times \vec{k}]$. The phenomenon described here also occurs during the diffraction reflection of antiprotons from a

crystal's surface and is caused by the interference between the spin-orbit amplitude and the real part of the Coulomb amplitude in the case when crystal's cells lack the center of symmetry. (Compare with a similar phenomenon for slow neutrons [37], in which case the effect occurs through the interference of the spin-independent part of nuclear scattering and the amplitude of spin-orbit scattering.)

Now, let us suppose that a particle beam is incident on the boundary between vacuum and matter with polarized nuclei. The elastic scattering amplitude $\hat{f}(\vec{k}' - \vec{k})$ in this case depends on spin orientations of the incident particle, \vec{S} , and the nucleus, \vec{J} , i.e., the scattering amplitude is the operator in the spin space of particle and nucleus. Investigating refraction and mirror reflection, we are interested in coherent elastic scattering, in which the nuclear spin state remains unchanged. The scattering amplitude $\hat{f}_N(\vec{k}' - \vec{k})$, describing such scattering, is obtained by averaging the total amplitude \hat{f} using the nuclear spin density matrix $\hat{\rho}_J$: $\hat{f}(\vec{k}' - \vec{k}) = \text{Tr } \hat{\rho}_J \hat{f}(\vec{k}' - \vec{k})$. (The general expression for the amplitude \hat{f} of scattering of a particle with spin $S = \frac{1}{2}$ by a nucleus with spin $J = \frac{1}{2}$ is given, e.g., in [6].)

Consequently, in this case the contribution from nuclear scattering to the amplitude of a mirror-reflected wave can be written in the form:

$$\hat{F}_{\text{pol}} = -\frac{\pi\rho}{k_z^2} \hat{f}_N(\vec{k}' - \vec{k}) \quad (115)$$

As a result, the amplitude of a mirror-reflected wave can be presented in the form

$$\hat{F}(\vec{k}' - \vec{k}) = F_{\text{Coul}}(\vec{k}' - \vec{k}) + \hat{F}_{so}(\vec{k}' - \vec{k}) + \hat{F}_{\text{pol}}(\vec{k}' - \vec{k}), \quad (116)$$

where F_{Coul} is the amplitude of the mirror-reflected wave that is due to Coulomb interaction, $\hat{F}_{so}(\vec{k}' - \vec{k})$ is the amplitude of the reflected wave that is due to antiproton–nucleus (or antiproton–atom) spin-orbit interaction, and $\hat{F}_{\text{pol}}(\vec{k}' - \vec{k})$ is the amplitude of antiproton scattering by a polarized nucleus (except for those terms in the amplitude \hat{F}_{pol} that describe spin-orbit interactions).

Using (116), we can find the intensity and polarization of reflected particles. For example, the intensity of reflected particles is related to spin orientation of incident particles by the expression of the form

$$I_{\text{ref}} = I_0 \text{Tr } F \rho_0 F^+ = I_0 \text{Tr } F^+ F \rho_0, \quad (117)$$

where ρ_0 is the spin density matrix of the incident beam and I_0 is the beam's intensity.

The polarization vector \vec{p} of mirror-reflected particles has the form

$$\vec{p} = \frac{1}{I_{\text{ref}}} \text{Tr } \rho_0 F^+ \frac{\hat{S}}{S} F, \quad (118)$$

where \hat{S} is the spin operator of particles; the spin of antiprotons equals 1/2, hence $\hat{S} = \frac{1}{2}\vec{\sigma}$.

It follows from (116), (117), and (118) that I_{ref} and \vec{p} depend on the interference of the nuclear amplitude $\hat{F}_{\text{pol}}(\vec{k}' - \vec{k})$ and the Coulomb and spin-orbit amplitudes.

The amplitude \hat{f}_N that determines the reflection amplitude \hat{F}_{pol} has quite a complicated structure. For slow antiprotons scattered at the angle $\vartheta \ll 1$ (the grazing angle $\varphi \ll 1$), the amplitude \hat{f}_N coincides with a zero-angle scattering amplitude and has the form

$$\hat{f}_N = A_0 + A_1(\vec{S} \vec{p}_t) + A_2(\vec{S} \vec{e})(\vec{e} \vec{p}_t), \quad (119)$$

where \vec{p}_t is the polarization vector of the target. Using (117), (118), and (119), we can find the intensity and polarization of reflected particles for each particular case. These expressions yield that when nonpolarized antiprotons are incident on a polarized target, the reflected antiprotons appear to be polarized. If the incident antiproton beam is polarized, then the spin of the mirror-reflected beam undergoes rotation (the rotation angle is estimated at the order of $10^{-1} \div 10^{-2}$), and the intensity of the reflected beam depends on the mutual orientation between the spin of incident particles and the target polarization. The degree of polarization that the initially nonpolarized beam acquires through reflection has the order of magnitude about $10^{-1} \div 10^{-2}$.

5. Quasi-optical Phenomena for High-energy Particles in Polarized Matter

For thermal neutrons, $v = 2.2 \cdot 10^5 \text{ cm}\cdot\text{s}^{-1}$; thus, a full spin turn occurs at length $L \approx 10^{-3} \text{ cm}$. The velocity increases with the energy growth approaching the speed of light c . Therefore, length L , at which a full spin turn occurs, increases 10^5 times and can be as long as 100 cm. With the growth of particle energy, the nuclear spin phenomenon becomes available for positively charged particles and nuclei (Coulomb repulsion does not prevent nuclear interactions in this case).

Now we shall pass on to the consideration of the effects of “optical” spin rotation arising when high-energy particles pass through matter with polarized nuclei [88–90].

To be more specific, we shall first consider refraction of relativistic particles with spin 1/2 in matter. In the beginning, let us consider scattering by a particular center. The asymptotic expression for a particle wave function far from the scatterer can be represented in the form [15, 62]

$$\Psi = U_{E\vec{k}} e^{ikz} + U'_{E'\vec{k}'} \frac{e^{ik'r}}{r}, \quad (120)$$

where $U_{E\vec{k}}$ is the bispinor amplitude of the incident plane wave; $U'_{E'\vec{k}'}$ is the bispinor describing the amplitude of the scattered wave; E and $\vec{k}(E', \vec{k}')$ are the energy and the wave vector of the incident (scattered) wave.

According to [62], the bispinor amplitude is fully determined by specifying a two-component quantity — a three-dimensional spinor W , which is a nonrelativistic wave function in the particle rest frame. For this reason, the scattering amplitude, i.e., the amplitude of a divergent spherical wave, in (120) like that in the nonrelativistic case, can be defined as a two-dimensional matrix of \hat{f} by the relation $W' = \hat{f}W$, where W' is the spinor determining the bispinor $U'_{E'\vec{k}'}$. The scattering operator thus determined is quite similar to the scattering amplitude in the nonrelativistic scattering theory.

As a result, by deriving the expression for the refractive index by analogy with the nonrelativistic case, we obtain the following expressions for the wave function of a relativistic neutron (proton) in a medium

$$\Psi = \frac{1}{\sqrt{2E}} \begin{pmatrix} \sqrt{E+m} e^{ik\hat{n}z} & W \\ \sqrt{E-m}(\vec{\sigma}\vec{n}) e^{ik\hat{n}z} & W \end{pmatrix}, \quad (121)$$

where

$$\hat{n} = 1 + \frac{2\pi\rho}{k^2} \hat{f}(0) \quad (122)$$

is the operator refractive index, $\hat{f}(0)$ is the amplitude of coherent elastic zero-angle scattering by a polarized scatterer; $\vec{\sigma}$ is the vector made up of the Pauli matrices; $\vec{n} = \vec{k}/k$.

Using $\vec{\sigma}$, \vec{J} , \vec{n} (\vec{J} is the nuclear spin operator), we may write the amplitude of coherent elastic forward scattering of a particle with spin 1/2 ($\vec{S} = (1/2)\vec{\sigma}$) by a polarized nucleus in the general case of strong, electromagnetic, and P-, T-violating weak interactions as follows [23, 37, 86, 87]:

$$\hat{f} = \hat{f}_\mu + \hat{f}_s + \hat{f}_w, \quad (123)$$

where \hat{f}_μ is the amplitude of coherent elastic scattering by a nucleus through magnetic interaction. Its explicit form depends on the observation conditions due to a long-range character of the magnetic interaction. The scattering amplitude caused by parity-conserving strong interactions:

$$\hat{f}_s = A + A_1 \vec{\sigma} \langle \vec{J} \rangle + A_2 (\vec{\sigma} \vec{n}) (\vec{n} \langle \vec{J} \rangle) + A_3 \vec{n} \vec{n}_1 + \dots$$

and the contribution from weak interactions to the scattering amplitude

$$\hat{f}_w = B \vec{\sigma} \vec{n} + B_1 \vec{\sigma} [\langle \vec{J} \rangle \vec{n}] + B_2 \vec{\sigma} \vec{n}_1 + B_3 (\vec{\sigma} \vec{n}) (\vec{n} \vec{n}_1) + B_4 \vec{n} \langle \vec{J} \rangle + B_5 \vec{\sigma} [\vec{n} \vec{n}_1] + \dots,$$

where $\langle \vec{J} \rangle = \text{Tr} \rho_{\text{nuc}} \vec{J}$ is the average nuclear spin, ρ_{nuc} is the spin density matrix of the nucleus, \vec{n}_1 is the vector with components $n_{1i} = \sum_{j=1}^3 \langle Q_{ij} \rangle n_j$, $\langle Q_{ij} \rangle = \text{Tr} \rho_{\text{nuc}} Q_{ij}$ is the target quadrupolarization tensor of rank two and

$$Q_{ij} = \frac{1}{2J(2J-1)} J_i J_j + J_j J_i - \frac{2}{3} J(J+1) \delta_{ij},$$

the sign $+\dots$ means the contributions of higher-order multipoles to \hat{f} .

Where the terms containing A are caused by strong P-, T-even interactions, those with B , B_2 , B_3 , B_4 — by P-odd T-even, B_1 — by P-odd T-odd and B_5 corresponds to P-even T-odd interactions.

The terms proportional to A_1 and A_2 give rise to a phenomenon of nuclear precession of the particle spin caused by a nuclear “pseudomagnetic” field. The terms proportional to B_i also lead to spin precession. The constant B describes the phenomenon of P-violating particle spin rotation about the momentum direction, predicted by Michel [19], and P-odd spin dichroism [20]. The constant B_1 [86, 87] describes the phenomenon of P- and T-odd spin rotation and dichroism. It should be pointed out that, unlike the term proportional to B_1 , the contributions to \hat{f} proportional to B_2 , B_3 , B_5 [86, 87] include the second-rank nuclear spin operator. The terms proportional to constants B_2 and B_3 describe P-odd T-even particle spin rotation and dichroism, while the term proportional to constant B_5 describes T-odd P-even particle spin rotation and dichroism.

According to (121), the spinor W' defining the spin state of a particle after passing the path length z in matter has the form

$$W' = e^{ik\hat{n}z} W. \quad (124)$$

Note that \hat{n} can be written as

$$\hat{n} = n_0 + \frac{2\pi\rho}{k^2} (\vec{\sigma} \vec{g}), \quad (125)$$

where n_0 is the $\vec{\sigma}$ -independent part of n .

From expression for $\hat{f}(0)$ we have

$$n_0 = 1 + \frac{2\pi\rho}{k^2} (A + A_3 \vec{n} \vec{n}_1 + B_4 \vec{n} \langle \vec{J} \rangle + \dots). \quad (126)$$

The explicit form of vector \vec{g} immediately follows from the expression for the amplitude:

$$\vec{g} = A_1 \langle \vec{J} \rangle + A_2 \vec{n} (\vec{n} \langle \vec{J} \rangle) + B \vec{n} + B_1 [\langle \vec{J} \rangle \vec{n}] + \dots \quad (127)$$

Suppose that particle absorption can be neglected; as a consequence, \vec{g} is a real vector. With the help of (125), represent (124) as follows:

$$W' = e^{ikn_0 z} e^{i \frac{2\pi\rho}{k} (\vec{\sigma} \vec{J}_g) |\vec{g}| z} W, \quad (128)$$

$\vec{j}_g = \vec{g}/|\vec{g}|$. Remember now that the operator of spin rotation by an angle ϑ about a certain axis characterized by a unit vector \vec{j} has the form

$$\hat{T} = e^{i \frac{\vartheta}{2} \vec{\sigma} \vec{j}}. \quad (129)$$

Comparing (124), (125) and (128), we obtain that in the case under consideration the operator

$$\exp \left\{ i \frac{2\pi\rho}{k} |\vec{g}| (\vec{\sigma} \vec{j}_g) z \right\}$$

acts as a spin rotation operator of a particle in its rest frame. The rotation angle is

$$\vartheta = \frac{4\pi\rho}{k} |\vec{g}| z = k(n_{\uparrow\uparrow} - n_{\downarrow\uparrow})z, \quad (130)$$

where the quantization axis is selected along \vec{j}_g .

5.1. The effect of spin precession at high energies

To be more specific, we shall further analyze the effect of spin precession in a polarized target due to strong interactions. In (121), (130), for the angle of particle spin rotation about the direction \vec{j}_g we obtain

$$\vartheta = \frac{2\pi\rho}{k} (f_{\uparrow\uparrow} - f_{\uparrow\downarrow})z = \frac{4\pi\rho}{k} |A_1 \langle \vec{J} \rangle + A_2 \vec{n} (\vec{n} \langle \vec{J} \rangle)|z. \quad (131)$$

To answer a question about how the spin precession angle ϑ of a relativistic particle in a polarized target depends on the particle energy, let us remember that when scattering by a potential, the Dirac equation for ultrarelativistic particles reduces to an equation similar to a non-relativistic Schrödinger equation, where the particle mass M stands for its relativistic mass [62], i.e., $M = \gamma m$, where m is the particle rest mass, γ is its Lorentz factor. Since the amplitude of particle scattering by a potential is proportional to the particle mass, the amplitude for a relativistic particle may be written as

$$f(E, 0) = \gamma f'(E, 0), \quad (132)$$

where E is the particle energy.

Such a relation also holds for the general case of scattering of a relativistic particle by a scatterer (for example, by a nucleus) [15]. According to [15], the forward scattering amplitude is related to the \mathcal{T} -matrix describing the collision of particles in the general case as follows

$$f(E, 0) = -(2\pi)^2 \frac{m\gamma}{\hbar^2} \mathcal{T}(E). \quad (133)$$

From this, we have

$$f'(E, 0) = -(2\pi)^2 \frac{m}{\hbar^2} \mathcal{T}(E). \quad (134)$$

Using (132), we can rewrite (131) as follows

$$\vartheta = 2\pi\rho\lambda_C(f'_{\uparrow\uparrow} - f'_{\downarrow\uparrow})z, \quad (135)$$

where $\lambda_C = \hbar/(mc)$ is the Compton wavelength of the particle.

By means of the \mathcal{T} -matrix, the expression for the rotation angle ϑ may also be represented in the form:

$$\vartheta = -\frac{(2\pi)^3\rho}{\hbar c}(\mathcal{T}_{\uparrow\uparrow}(E) - \mathcal{T}_{\downarrow\uparrow}(E))z. \quad (136)$$

The particle path length z in the target is $z = vt$, v is the particle velocity, t is the time in which the particle passed the path z . Hence, we also have $\vartheta = \omega_{\text{pr}}t$, where ω_{pr} is the particle spin precession frequency in a polarized target

$$\omega_{\text{pr}} = -\frac{(2\pi)^3\rho}{\hbar c}(\mathcal{T}_{\uparrow\uparrow}(E) - \mathcal{T}_{\downarrow\uparrow}(E))v.$$

As we see, at relativistic energies, the dependence $\vartheta \sim 1/k$ disappears, and the entire possible dependence of the rotation angle ϑ on the particle energy is contained in the amplitude $f'(E, 0)(\mathcal{T}(E))$.

Evaluate the possible rotation angle (see also [90]). To understand the magnitude of the effect, let us assume that the difference $f'_{\uparrow\uparrow} - f'_{\downarrow\uparrow}$ is of the order of $10^{-12} \div 10^{-13}$ cm, i.e., of the order of the difference between the spin amplitudes of neutron scattering by a proton in a low-energy range. In this case, for a fully polarized target, we have

$$\vartheta \sim 10^{-3} \div 10^{-4}z, \quad (137)$$

where ϑ is in radians, i.e., $\vartheta \sim 10^{-2} \div 10^{-3}$ rad for a particle passing through a polarized target of length 10 cm.

The acquired $|p_x|$ values for a beam of polarized protons in a polarized external target at the Nuclotron are given in Table 1 for NH_3 and ND_3 targets. Thickness for both targets is as high as $L = 30$ cm, the proton energy is within 200 MeV–1 GeV range. Here $\Delta f = f_{\uparrow\uparrow} - f_{\downarrow\uparrow}$ and $\Delta\sigma = \sigma_{\uparrow\uparrow} - \sigma_{\downarrow\uparrow}$ denotes difference in the proton total scattering cross-section values for parallel and antiparallel spin orientations.

Table 1. Effect evaluation for polarized protons in polarized external targets at the Nuclotron.

Target	N_b	σ , b	L , cm	$\frac{\Delta\sigma}{\sigma} = \frac{\sigma_{\uparrow\uparrow} - \sigma_{\downarrow\uparrow}}{\sigma}$	$\frac{\text{Re}(\Delta f)}{\text{Im}(\Delta f)}$	$ p_x \approx \vartheta$
NH_3	10^{10}	0.5	30	0.02	0.5	$2 \cdot 10^{-3}$
ND_3	10^{10}	0.6	30	0.04	0.5	$4 \cdot 10^{-3}$

5.2. Spin Rotation and Spin Dichroism

With the increase of target thickness, the influence of scattering and absorption on the polarization characteristics of a particle transmitted through matter grows. Let, however, the target thicknesses be such that the spin-dependent contributions to the wave-function phase

are small, i.e., the inequalities $k\text{Re}gz \ll 1$ and $k\text{Im}gz \ll 1$ (see (127)) are valid. In this case, we can make use of the above results in order to find the polarization characteristics of the beam. The spinor W' defining the spin state of a particle after passing the path length z in matter has the form

$$W' = e^{ik\hat{n}z} W \simeq e^{ikn_0z} \left(1 + i \frac{2\pi\rho}{k} (\vec{\sigma}\vec{g})z \right) W. \quad (138)$$

From (138) follows the expression for the number of particles N transmitted through the target in the same direction as the direction of the momentum of the particles N_0 incident on the target:

$$N = N_0 e^{-\rho\sigma z} \left[1 - \frac{4\pi\rho}{k} \vec{P}_0 \text{Im}\vec{g}z \right], \quad (139)$$

where σ is the spin-independent part of the total scattering cross section determined by the imaginary part of n_0 in (126)

$$\sigma = \frac{4\pi}{k} \text{Im}(A + A_3 \vec{n}\vec{n}_1 + \dots), \quad (140)$$

\vec{P}_0 is the particle polarization vector before entering the target, \vec{g} is defined by (127).

According to (139), the number of particles N transmitted through the target depends on the orientation of \vec{P}_0 : $N_{\uparrow\uparrow} \neq N_{\downarrow\uparrow}$, where $N_{\uparrow\uparrow}$ describes N for $\vec{P}_0 \uparrow\uparrow \text{Im}\vec{g}$, $N_{\downarrow\uparrow}$ denotes N for $\vec{P}_0 \downarrow\uparrow \text{Im}\vec{g}$.

Thus, spin dichroism occurs because the absorption coefficient of the incident particles in the target depends on the orientation of their spin.

Using (138) for the spin wave function W' , we obtain the following expression for the polarization vector \vec{P} of the particles transmitted through a polarized target:

$$\vec{P} = \frac{\langle W' | \vec{\sigma} | W \rangle}{\langle W' | W \rangle} = \vec{P}_0 + \frac{4\pi\rho z}{k} \text{Im}((\vec{P}_0\vec{g})\vec{P}_0 - \vec{g})z + \frac{4\pi\rho z}{k} [\vec{P}_0 \times \text{Re}\vec{g}]. \quad (141)$$

In view of (141), the polarization vector of high-energy particles, like that of low-energy particles, undergoes rotation about the direction of $\text{Re}\vec{g}$. In a similar manner as in the case of low energies, the contributions associated with strong and P-, T-odd weak interactions can be distinguished by measuring the magnitudes of N and \vec{P} for different orientations of the polarization vector \vec{P}_0 of the particles incident on the target.

It is worth mentioning that for nuclei with spin $J \geq 1$, in the cross-section σ the term $\text{Im}A_3 \vec{n}\vec{n}_1$ determined by the target quadrupolarization is different from zero.

Suppose that a nonpolarized beam of particles ($\vec{P}_0 = 0$) is incident on a target (e.g. a beam of protons is incident on a deuterium target). This situation is typical for the present-day experiments with protons (antiprotons) and a deuterium gas target in a storage ring or with beams in a collider, for example, NICA.

Let us choose the quantization axis along the direction \vec{n} of the beam momentum at the target location. We have (see (140))

$$\sigma = \frac{4\pi}{k} (\text{Im}A + \text{Im}A_3 Q_{zz}^D), \quad (142)$$

where Q_{zz}^D is the quadrupolarization tensor of the deuteron of the target.

Let the deuterons be in spin state with a magnetic quantum number $M \pm 1$.

In this case $Q_{zz}^D = \frac{1}{3}$.

But if the deuteron is in spin state $M = 0$, then $Q_{zz}^D = -\frac{2}{3}$. Consequently, the below equalities may be written for the corresponding total interaction cross sections:

$$\begin{aligned}\sigma_{\pm 1} &= \sigma_{\text{non}} + \frac{4\pi}{3k} \text{Im}A_3, \\ \sigma_0 &= \sigma_{\text{non}} - \frac{4\pi}{k} \frac{2}{3} \text{Im}A_3,\end{aligned}\quad (143)$$

σ_{non} is the total scattering cross section of a nonpolarized proton by a nonpolarized deuteron, i.e.,

$$\text{Im}A_3 = \frac{k}{4\pi} (\sigma_{\pm 1} - \sigma_0). \quad (144)$$

According to (139), the number $N(\pm 1)$ of protons (antiprotons) transmitted through a polarized target with deuterons in spin state ($M = \pm 1$) is

$$N(\pm 1) = N_0 e^{-\rho\sigma_{\pm 1}z}, \quad (145)$$

while in the case when $M = 0$, it is

$$N(0) = N_0 e^{-\rho\sigma_0z}. \quad (146)$$

In view of (139), (145), (146), the absorption of protons (antiprotons) transmitted through the target will be different for different orientations of the tensor polarization for the deuteron target. Suppose now that a beam of nonpolarized particles (protons, antiprotons, deuterons, nuclei) moves in a storage ring. It is obvious that since the absorption coefficient of the particles depends on the orientation of the tensor polarization for the deuteron target, the number of particles moving in a storage ring will also depend on this characteristic of the target. In other words, the lifetime of a nonpolarized beam in a storage ring depends on the orientation of the tensor polarization for the deuteron target [92, 93]. As a result, by measuring the beam lifetime in this case, one can determine $\text{Im}A_3$, i.e., a spin-dependent part of the total cross-section of proton (antiproton, deuteron) scattering by a polarized deuteron, which is proportional to Q_{zz} .

5.3. Proton (Antiproton) Spin Rotation in a Thick Polarized Target and Spin Filtering of Particle Beams in a Nuclear Pseudomagnetic Field

With further increase in the target thickness, the influence of the spin-dependent part of particle absorption in matter is enhanced.

In order to obtain equations describing the evolution of intensity and polarization of a beam in the target, we shall split vector \vec{g} into real and imaginary parts:

$$\vec{g} = \vec{g}_1 + i\vec{g}_2, \quad (147)$$

where $\vec{g}_1 = \text{Re}\vec{g}$ and $\vec{g}_2 = \text{Im}\vec{g}$.

Using (124), (125), one may obtain the following system of equations defining the relation between the number of particles transmitted through the target $N(z)$ and their polarization $\vec{P}(z)$ [90]:

$$\frac{d\vec{P}(z)}{dz} = \frac{4\pi\rho}{k} [\vec{g}_1 \times \vec{P}(z)] - \frac{4\pi\rho}{k} \left\{ \vec{g}_2 - \vec{P}(z)[\vec{g}_2 \vec{P}(z)] \right\}. \quad (148)$$

Acting analogously, we can obtain the equation for the change in the polarized beam intensity in a polarized nuclear target:

$$\frac{dN(z)}{dz} = -2\text{Im}(n_0)kN(z) - \frac{4\pi\rho}{k} [\vec{g}_2 \vec{P}(z)]N(z). \quad (149)$$

(148) and (149) should be solved together under the initial conditions $\vec{P}(0) = \vec{P}_0$, $N(0) = N_0$. According to (148), the polarization of the incident particles passing through the polarized nuclear target undergoes rotation through the angle

$$\theta = \frac{4\pi\rho}{k} g_1 z. \quad (150)$$

Assume that the target polarization $\vec{P}_t = \langle \vec{J} \rangle / J$ is parallel (this is the case of longitudinal polarization) or orthogonal (transversal polarization) to the momentum of the incident particle \vec{k} . Then $\vec{g}_1 \parallel \vec{g}_2 \parallel \vec{P}_t$ and (148) and (149) are reduced to a simple form.

Consider two specific cases for which the initial polarization of the incident beam is (a) $\vec{P}_0 \parallel \vec{P}_t$ or (b) $\vec{P}_0 \perp \vec{P}_t$.

Case (a) is a standard transmission experiment, in which we observe the process of absorption in the polarized target without a change in the direction of the initial beam polarization. The absorption is different for particles polarized parallel and antiparallel to the target polarization. The number of particles N changes according to

$$N(z) = N_0 \exp(-\sigma_{\pm} \rho z), \quad (151)$$

where

$$\sigma_{\pm} = \frac{4\pi}{k} [\text{Im}(A + A_3 \vec{n} \vec{n}_1) \pm \text{Im}A_1 J P_t \pm \text{Im}A_2 J P_t]. \quad (152)$$

In case (b), the coherent scattering by the polarized nuclei results in spin rotation of the incident particles about the target polarization \vec{P}_t .

According to (131), the spin rotation angle

$$\vartheta = \frac{4\pi\rho}{k} \text{Re}g = \frac{4\pi\rho}{k} \left[\text{Re}A_1 J P_t + \text{Re}A_2 J (\vec{n} \vec{P}_t) \right] z \quad (153)$$

is directly connected with the real part of the forward scattering amplitudes. The values $\text{Re}A_1$ and $\text{Re}A_2$ can be determined separately by measuring spin rotation angles for two cases, when the target spin is parallel and antiparallel to the beam direction \vec{n} . This means that by measuring the final intensity and polarization of the beam in cases (a) and (b), we can directly reconstruct the spin dependent forward scattering amplitude.

In particular, as shown in [90], the measurement of the rotation angle near resonances or near the threshold values at which inelastic reaction channels open enables the apprehension of the nature of resonances and the investigation of the nature of threshold phenomena.

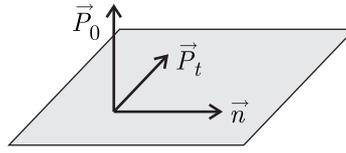


Figure 8. Beam polarization \vec{P}_0 is perpendicular to the plane formed by vectors \vec{P}_t and \vec{n} .

Let us assume now that the target polarization is directed at some angle (which does not equal $\pi/2$) with respect to the incident particle momentum and that the incident beam polarization is perpendicular to the plane formed by vectors \vec{P}_t and \vec{n} (see Fig. 8). In this case, the effect of proton (antiproton) spin rotation about vector \vec{g}_1 , combined with absorption dichroism determined by vector \vec{g}_2 , will cause the dependence of the total number of particles transmitted through the target on $\text{Re}A_1$ and $\text{Re}A_2$ [90, 94]:

$$N(z) \sim (\text{Re}A_1 \cdot \text{Im}A_2 - \text{Re}A_2 \cdot \text{Im}A_1) \left[\vec{P}_t \times \vec{n}(\vec{n}\vec{P}_t) \right] \vec{P}_0. \quad (154)$$

Such behavior of $N(z)$ enables measuring spin-dependent contributions to the amplitude $f(0)$ in the transmission experiment without measuring the polarization of the beam transmitted through the target [90, 94].

This implies that if a vertically polarized beam rotates in the storage ring (the beam's spin is orthogonal to the orbit plane), then the decrease in the beam intensity (the beam lifetime in the storage ring) in the case when vectors \vec{P}_t and \vec{n} lie in the horizontal plane (in the orbit plane) enables determining $\text{Re}A_1$ and $\text{Re}A_2$.

Note that the contribution under study reverses sign when vector \vec{P}_t rotates about \vec{n} through π (or \vec{P}_0 changes direction: $\vec{P}_0 \rightarrow -\vec{P}_0$). By measuring the difference between the beam's damping times for these two orientations of \vec{P}_0 (\vec{P}_t), one can find the contribution of $\text{Re}A_1$ and $\text{Re}A_2$ to the real part of the amplitude $f(0)$.

Unlike a spin rotation experiment, this transmission experiment does not allow one to determine $\text{Re}A_1$ and $\text{Re}A_2$ separately.

There is also an inverse process: if a nonpolarized beam of particles is incident on the target, then after passing through the target, the beam acquires polarization orthogonal to the plane formed by \vec{P}_t and \vec{n} . This phenomenon is particularly interesting in the case of a storage ring. If \vec{P}_t and \vec{n} lie in the orbit plane of a circulating beam (i.e., the horizontal plane), then over time the beam acquires vertical polarization, orthogonal to the orbit plane. By measuring the arising vertical polarization, it is also possible to determine the spin-dependent part of the coherent elastic zero-angle scattering amplitude. In particular, in spin-filtering experiments for obtaining polarized beams of antiprotons (protons) [95–103], it is sufficient to perform an experiment under the conditions when the polarization vector \vec{P}_t of a gas target is directed at a certain angle, which is not equal to 0, π or $\pi/2$, with respect to the particle momentum direction \vec{n} . If $\text{Re}A_{1,2} \sim \text{Im}A_{1,2}$, the degree of arising polarization is comparable to the anticipated degree of polarization of the anti(proton) beam arising in the method [95–103], and enables one to measure the contribution proportional to $\text{Re}A_{1,2}$.

It is worth mentioning that in the case under consideration, the beam lifetime depends on the orientation of \vec{P}_t in the \vec{P}_t, \vec{n} -plane. In this case, the measurement of the beam lifetime also provides information about $\text{Re}A_{1,2}$.

Let us note that the terms similar to those proportional to A_1 and A_2 in the expression for the zero-angle scattering amplitude also appear in the expression for the scattering amplitude

of particles with spin $S \geq 1$ (nuclei). Hence, the phenomena described above exist for particles with spin $S \geq 1$ as well [23, 37].

5.4. Spin density matrix formalism

Under conditions when multiple scattering appears important, the quantum mechanical description of a beam of polarized particles traveling through a polarized target makes use of the spin density matrix w .

In our case, we treat the polarized target as a thermal reservoir with an infinite set of degrees of freedom, which we then average. Therefore, the density matrix for the system, $\hat{\rho}(b, T, t)$, consisting of the polarized target plus polarized particle beam, can be written as a direct product of the matrices $\hat{\rho}_b(t)$ and $\hat{\rho}_T(t)$ ($\hat{\rho}_b(t)$ is the beam spin density matrix, $\hat{\rho}_T(t)$ is the target spin density matrix):

$$\hat{\rho}(b, T, t) = \hat{\rho}_b(t) \otimes \hat{\rho}_T(t). \quad (155)$$

In accordance with the general theory [104], the master equation for the spin density matrix of above system can be expressed in a form [37, 73, 90, 91] as follows:

$$\frac{d\hat{\rho}_b(t)}{dt} = -\frac{i}{\hbar}(\hat{H}\hat{\rho}_b - \hat{\rho}_b\hat{H}) + \left(\frac{d\hat{\rho}}{dt}\right)_{\text{sct}} \quad (156)$$

with the Hamiltonian \hat{H} including the interaction between particles of the beam and external electromagnetic fields. The term $(d\hat{\rho}/dt)_{\text{sct}}$ describes the change in the density matrix due to collisions in the target.

A detailed account of general methods for obtaining the explicit form of the collision integral is given in many course books [15, 104]. These methods can also be applied to the consideration of the interaction between polarized beams and a polarized target [90, 91].

Let us consider the process of particle passage through a target consisting of N_T particles interacting with one another by means of a certain potential U .

Suppose that an incident particle (proton, antiproton, deuteron, nucleus) has a rest mass m and spin S_b . The target consists of N_T bound particles with mass M and spin s .

The Hamiltonian of the scattering system is written in the form:

$$H_T = \sum_{\alpha=1}^{N_T} K_{\alpha} + U, \quad (157)$$

where K_{α} is the kinetic energy operator of particle α , U is the interaction energy of N_T particles of the scatterer.

The solution of the Schrödinger equation

$$H_T\Psi_n = W_n\Psi_n \quad (158)$$

determines the possible values of the energy W_n of the system and the corresponding set of wave functions

$$\Psi_n = \Psi_n(\vec{R}_1, s_1, \dots, \vec{R}_N, s_N), \quad (159)$$

where $(\vec{R}_{\alpha}, s_{\alpha})$ are the spatial and spin coordinates of particle α in the target.

The operator of interaction between the incident particle and the target is defined in terms of V : $V = \sum_{\alpha=1}^{N_T} V_{\alpha}$. Here V_{α} describes the interaction between the beam particle and the target particle α .

The Hamiltonian of the whole system has the form:

$$H = K_b + H_T + V, \quad (160)$$

where K_b is the kinetic energy operator of the beam particle b .

To describe the process of particle b transmission through the target, find the density matrix $\hat{\rho}(t)$ of the system “incident particle + target”. This density matrix satisfies the quantum Liouville equation [104]:

$$i \frac{\partial \hat{\rho}(t)}{\partial t} = [\hat{H}, \hat{\rho}(t)]. \quad (161)$$

The solution of this equation can formally be written using the evolution operator $\hat{U}(t', t)$

$$\hat{\rho}(t') = \hat{U}(t', t) \hat{\rho}(t) \hat{U}^+(t', t), \quad (162)$$

which is related to the explicitly time-independent Hamiltonian of the system as $\hat{U}(t', t) = e^{-\frac{i}{\hbar} \hat{H}(t'-t)}$.

Let us consider the target as a thermostat ($N_T \gg 1$). Then the statistical operator $\hat{\rho}$ of the system can be represented as a direct product $\hat{\rho} = \hat{\rho}_b \otimes \hat{\rho}_T$, where $\hat{\rho}_b$ is the density matrix of the incident particle, $\hat{\rho}_T$ is the equilibrium density matrix of the medium. The equilibrium density matrix $\hat{\rho}_T$ of the target is diagonal in the stationary states n of the target: $\langle n | \hat{\rho}_T | n' \rangle = \delta_{nn'} \cdot \hat{\rho}_{nn'} = \delta_{nn'} \hat{\rho}(n)$. (Note that $\hat{\rho}_T(n)$ is the operator in the spin space of a nuclear target). The spin density matrix $\hat{\rho}_b$ includes elements that are diagonal and nondiagonal with respect to the momenta \vec{k} and \vec{k}' of the particle: $\hat{\rho}_b = \hat{\rho}_b(\vec{k}, \vec{k}')$. However, the nondiagonal elements oscillate rapidly, and after several collisions with target nuclei, in order to describe the process of multiple scattering, one can assume that the density matrix $\hat{\rho}_b(\vec{k}, \vec{k}')$ is diagonal [90, 91, 104–106], i.e., $\hat{\rho}_b(\vec{k}, \vec{k}') = \delta_{\vec{k}, \vec{k}'} \hat{\rho}_b(\vec{k})$. We shall note here that the particle spin density matrix $\hat{\rho}_b(\vec{k})$ is nondiagonal in the spin states describing a particle incident on the target.

The time interval Δt , during which the density matrix $\hat{\rho}_b(\vec{k}, \vec{k}')$ is diagonalized, satisfies the inequality $\Delta t \gg R/\bar{v}$, where R is the radius of action of the forces, \bar{v} is the particle mean velocity in matter. We shall further concern ourselves with the behavior of the system over the time interval $\Delta t \gg \frac{R}{\bar{v}}$. This means that the evolution operator $U(t', t)$ can be replaced by the Heisenberg’s S -matrix $S \equiv \lim_{t' \rightarrow \infty, t \rightarrow -\infty} U(t', t)$, which relates the asymptotic states of the system before scattering to those after scattering [90, 91, 104–106]. The matrix elements of the S -matrix for a scattering system consisting of N_T particles are defined as follows:

$$S_{fa} = \delta_{fa} - i(2\pi) \delta(E_f - E_a) \sum_{\alpha=1}^{N_T} (\mathcal{T}_{\alpha})_{fa}, \quad (163)$$

where E_a and E_f are the total energies of the system before and after scattering, respectively; $E_a = \varepsilon_k + W_n$, $E_f = \varepsilon_{k'} + W_{n'}$, ε_k and $\varepsilon_{k'}$ are the energies of the incident particle before and after the collision; \mathcal{T}_{α} is the scattering matrix of particle b interacting with a free particle α . We shall make use of the fact that in the case of high-energy particles, the energies of the particles are much greater than the binding energies of the scatterers in the target. This enables using impulse approximation [15].

In the impulse approximation, the scattering matrix \mathcal{T}_α coincides with the matrix of scattering of particle b by a free particle α [15]. Expression (162) can be rewritten for the density matrix of the incident particle, using the S -matrix defined in (163):

$$\hat{\rho}_b(t + \Delta t) = \text{Tr}_T S \hat{\rho}(t) S^+, \tag{164}$$

where Tr_T means taking the trace over the states of the target.

Write (164) for the elements of the density matrix which are diagonal in the momentum space:

$$\begin{aligned} \hat{\rho}_b(t + \Delta t, \vec{k}) &= \hat{\rho}_b(t, \vec{k}) - i(2\pi) \frac{\Delta t}{2\pi} \text{Tr}_T \sum_{\alpha=1}^{N_T} \langle \vec{k}, n | \hat{\mathcal{T}}_\alpha | \vec{k}, n \rangle \hat{\rho}(t, \vec{k}, n) \\ &+ i(2\pi) \frac{\Delta t}{2\pi} \text{Tr}_T \sum_{\alpha=1}^{N_T} \hat{\rho}(t, \vec{k}, n) \langle \vec{k}, n | \hat{\mathcal{T}}_\alpha^+ | \vec{k}, n \rangle \\ &+ (2\pi)^2 \frac{\Delta t}{2\pi} \text{Tr}_T \sum_{\alpha, \beta=1}^{N_T} \sum_{\vec{k}', n'} \langle \vec{k}, n | \hat{\mathcal{T}}_\alpha | \vec{k}', n' \rangle \hat{\rho}(t, \vec{k}', n') \\ &\times \delta(\varepsilon_{k'} - \varepsilon_k + W_{n'} - W_n) \langle \vec{k}', n' | \hat{\mathcal{T}}_\beta^+ | \vec{k}, n \rangle. \end{aligned} \tag{165}$$

Let us recall that $\hat{\rho}(t, \vec{k}', n) = \hat{\rho}_b(t, \vec{k}) \otimes \hat{\rho}_T(n)$. In the momentum space, the matrix elements for the scattering matrix operator $\hat{\mathcal{T}}_\alpha$ have the form [62]:

$$\langle \vec{k}', \vec{P}'_\alpha | \hat{\mathcal{T}}_\alpha | \vec{k}, \vec{P}_\alpha \rangle = (2\pi)^3 \delta(\vec{k}' + \vec{P}'_\alpha - \vec{k} - \vec{P}_\alpha) \langle \vec{k}', \vec{P}'_\alpha | \hat{\mathcal{T}}_\alpha | \vec{k}, \vec{P}_\alpha \rangle, \tag{166}$$

\vec{P}_α denotes the momenta of the α -th scatterer, $\hat{\mathcal{T}}_\alpha$ is the scattering matrix on the momentum shell. Recall that $\hat{\mathcal{T}}_\alpha$ and $\hat{\mathcal{T}}_\alpha^+$ remain the operators with respect to spin variables.

The states with a definite value of the momentum $|\vec{k}\rangle$ are normalized according to equality

$$\langle \vec{k}' | \vec{k} \rangle = \frac{(2\pi)^3}{V} \delta(\vec{k}' - \vec{k}),$$

where V is the normalization volume. Thus, substitution of summation over all \vec{k} by integration is made as follows

$$\sum_{\vec{k}} \rightarrow \frac{V}{(2\pi)^3} d^3 \vec{k};$$

the Kronecker symbol and the Dirac function δ are related as

$$\delta_{\vec{k}\vec{k}'} \rightarrow \frac{(2\pi)^3}{V} \delta(\vec{k} - \vec{k}').$$

Scattering processes that make the contribution to (165) can be both elastic and inelastic. Elastic scattering occurs in the absence of any excitation in the scatterer. Inelastic scattering accompanied by a small excitation in the scatterer is called “quasi-elastic” [15]. For such scattering the momentum \vec{q} transferred to the α -th scatterer is $q \ll \sqrt{2MK_\alpha}$, $q \ll \sqrt{2MU_\alpha}$, $\vec{q} = \vec{k}' - \vec{k}$; \vec{k} is the momentum of the incident particle before the collision with the scatterer, \vec{k}' is the momentum of the beam particle after the collision.

When the momentum transferred to the scatterer α exceeds the momentum of the target particle in the initial bound state $q \gg \sqrt{2MK_\alpha}$, $q \gg \sqrt{2MU_\alpha}$, then the so-called “quasi-free”

approximation can be used. In this approximation, the excitation energy of the system in one collision is exactly equal to the recoil energy $\vec{q}^2/2M$ of a free particle of the target. Note that in summation over different nuclei of the target, sums of the form given below appear in the last term in (165)

$$\sum_{\alpha,\beta=1}^{N_T} \int d^3\vec{R}_1 \dots d^3\vec{R}_N e^{-i\vec{q}(\vec{R}_\alpha - \vec{R}_\beta)} \hat{\rho}_T(\vec{R}_1, \dots, \vec{R}_N) = N_T \hat{\rho}_T + \sum_{\alpha \neq \beta}^{N_T} \int d^3\vec{R}_1 \dots d^3\vec{R}_N e^{-i\vec{q}(\vec{R}_\alpha - \vec{R}_\beta)} \hat{\rho}_T(\vec{R}_1, \dots, \vec{R}_N), \quad (167)$$

where $\hat{\rho}_T$ is the spin density matrix of the target nucleus. In deriving (167), it is assumed that the positions of target nuclei and their spin states are uncorrelated.

Upon averaging over the spin states of nuclei in the target, the second term in (167) can be expressed in terms of the so-called particle pair distribution function [15], and vanishes when the transferred momentum \vec{q} exceeds the magnitude inverse to the correlation radius r , i.e., $q \gg r^{-1}$ [15]. For a non-crystalline target, the magnitude of the correlation radius r is of the order of the distance between nuclei. Consequently, the second term can only contribute to the kinetic equation at very small scattering angles $\vartheta_{sc} \lesssim 1/kr$, so it will be neglected in further consideration.

The resulting expression for the density matrix can be obtained from (165) for

$$\frac{\hat{\rho}_b(t + \Delta t) - \hat{\rho}_b(t)}{\Delta t} = \frac{\Delta \hat{\rho}_b(t)}{\Delta t}.$$

The time interval Δt should be chosen to be much greater than the characteristic correlation time of the process (the time interval during which the density matrix is diagonalized), but still small enough to fulfil the condition that the difference $\Delta \hat{\rho}_b$ is linear over Δt .

Thus, the equation for the density matrix describing the behavior of the polarized proton (antiproton, deuteron, nucleus) beam in polarized matter can be written as:

$$\begin{aligned} \frac{d\hat{\rho}_b(\vec{k}, t)}{dt} &= -iV N_T \text{Tr}_T \left(\hat{\mathcal{T}}(\vec{k}, \vec{k}) \hat{\rho}(\vec{k}, t) - \hat{\rho}(\vec{k}, t) \hat{\mathcal{T}}^+(\vec{k}, \vec{k}) \right) \\ &+ \frac{V^3}{(2\pi)^2} N_T \text{Tr}_T \int d^3\vec{k}' \delta \left(\varepsilon_k - \varepsilon_{k'} - \frac{\vec{q}^2}{2M} \right) \\ &\times \hat{\mathcal{T}}(\vec{k}, 0; \vec{k}', -\vec{q}) \hat{\rho}(\vec{k}', t) \hat{\mathcal{T}}^+(\vec{k}', -\vec{q}; \vec{k}, 0). \end{aligned} \quad (168)$$

$\hat{\rho}(\vec{k})$ denotes the following dependence: $\hat{\rho}(\vec{k}) = \hat{\rho}_b(\vec{k}; \vec{S}_b) \otimes \hat{\rho}_T(\vec{S}_T)$, where $\hat{\rho}_T(\vec{S}_T)$ is the spin density matrix of the target nucleus.

Let us introduce the scattering amplitude \hat{F} with matrix elements equal to [62]:

$$\hat{F}(\vec{k}, \vec{k}') = -\frac{M_r}{2\pi} V^2 \hat{\mathcal{T}}(\vec{k}, 0; \vec{k}', -\vec{q}), \quad (169)$$

where

$$M_r = \frac{mM}{m + M}$$

is the reduced mass.

Then (168) can be transformed into a form:

$$\begin{aligned} \frac{d\hat{\rho}_b(\vec{k}, t)}{dt} &= \frac{2\pi i}{M_r} N_T \text{Tr}_T \left(\hat{F}(\vec{k}, \vec{k}) \hat{\rho}(\vec{k}, t) - \hat{\rho}(\vec{k}, t) \hat{F}^+(\vec{k}, \vec{k}) \right) \\ &+ N_T \text{Tr}_T \int d\Omega_{\vec{k}'} \frac{k'^2}{M_r^2 \left(\frac{k'}{m} - \frac{(\vec{k}-\vec{k}')\vec{n}'}{M} \right)} \hat{F}(\vec{k}, \vec{k}') \hat{\rho}(\vec{k}', t) \hat{F}^+(\vec{k}', \vec{k}), \end{aligned} \quad (170)$$

where N_T denotes the number of particles in the target per unit volume, \vec{n}' is the unit vector in the direction of the momentum \vec{k}' . The absolute value of vector \vec{k}' is determined from the equation:

$$\varepsilon_k = \varepsilon_{k'} + \frac{(\vec{k} - \vec{k}')^2}{2M}. \quad (171)$$

Note that when the condition $m \geq M$ for the masses of the incident particles is fulfilled, the denominator in the integrand of (170) tends to zero for the value of the scattering angle of the incident particle θ , which is defined by the equality:

$$\cos \theta = \frac{\sqrt{m^2 - M^2}}{m},$$

with the absolute value of vector \vec{k}'

$$\vec{k}' = k \sqrt{\frac{m - M}{m + M}}.$$

Expression (170) is simplified when a particle (proton, deuteron, antiproton) passes through a target with nuclei whose mass is much larger than the mass of the incoming particle. In this case, we can neglect the effect of the energy loss of the incident particle through scattering. So we can neglect the recoil energy $\vec{q}^2/2M$ in the δ -function (168), (171). As a result, one obtains a simple kinetic equation describing the time and spin evolution of the incident particle as it passes through the target [90, 91]:

$$\begin{aligned} \frac{d\hat{\rho}_b(\vec{k}, t)}{dt} &= \frac{2\pi N_T}{m} \text{Tr}_T \left[\hat{F}(\vec{k}, \vec{k}) \hat{\rho}_b(\vec{k}, z) - \hat{\rho}_b(\vec{k}, z) \hat{F}^+(\vec{k}, \vec{k}) \right] \\ &+ N_T \frac{k}{m} \text{Tr}_T \int d\Omega_{\vec{k}'} \hat{F}(\vec{k}, \vec{k}') \hat{\rho}(\vec{k}', t) \hat{F}^+(\vec{k}', \vec{k}), \end{aligned} \quad (172)$$

where $|\vec{k}| = |\vec{k}'|$.

The first term on the right-hand side of (172), which describes refraction of particle in the target, can be represented as follows:

$$\begin{aligned} \hat{F}(0) \hat{\rho}(\vec{k}, t) - \hat{\rho}(\vec{k}, t) \hat{F}^+(0) &= \\ \left[\frac{1}{2} \left(\hat{F}(0) + \hat{F}^+(0) \right), \hat{\rho}(\vec{k}, t) \right] &+ \left\{ \frac{1}{2} \left(\hat{F}(0) - \hat{F}^+(0) \right), \hat{\rho}(\vec{k}, t) \right\}, \end{aligned} \quad (173)$$

where $[..., ...]$ is the commutator, $\{ ..., ... \}$ is the anticommutator.

The part proportional to the commutator leads to the rotation of the polarization vector due to elastic coherent scattering (as a result of the refraction effect [90, 91]); the anticommutator describes the reduction in the intensity and polarization of the beam which has passed through

the target. The last term in (172) determines the effect of incoherent scattering on the change of $\hat{\rho}_b$ (in the general case, single and multiple scattering).

As stated above, (172) is not applicable to the description of the process of proton (deuteron) transmission through the target containing light nuclei (protons, deuterons). To describe multiple scattering in this case, a more general (170) should be solved.

As a result, it is possible to find the dependence of the intensity and the polarization characteristics of the beam on the direction of the particle scattering and on the distance z traveled by the particle in matter.

In a real experiment, the scattered particles are registered within a certain interval of finite momenta because the collimator of the detector has a finite angular width. Therefore, we should study the characteristics of the beam transmitted through the target in the interval of solid angles $\Delta\Omega$ with respect to the initial direction of the beam propagation.

In fact, due to the axial symmetry of the collimator, $\Delta\Omega$ is determined by the angular width $2\vartheta_{\text{det}}$ of the detector collimator.

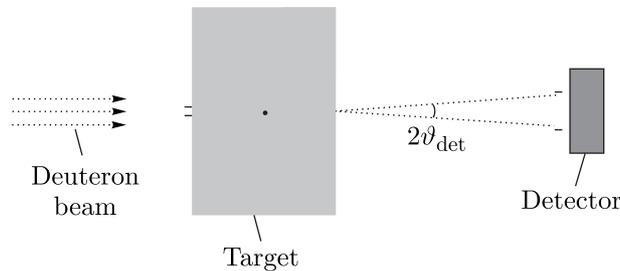


Figure 9. Scheme of transmitted beam detection by the central detector.

Equation (172) was analyzed in [90, 91, 106], where two physical mechanisms of spin rotation were indicated: one due to refraction of particles in a polarized target (“optical” spin rotation) (the second term in (172)) and the other, appearing as a result of spin rotation through incoherent scattering and caused by Coulomb–nuclear interaction and incoherent scattering by nuclei [37, 90, 106–108] (the third term in (172) (see [91, 108])). As shown in [108], using different angular resolutions $\Delta\Omega$ of the detector, one can study different contributions to spin rotation.

Thus, multiple scattering does not cancel the effect of spin rotation of charged particles in a polarized target.

6. The phenomenon of birefringence (spin oscillation and spin dichroism) of particles with spin $S \geq 1$

In the previous sections, we have found out that the refraction of particles in matter with polarized nuclei leads to “optical” spin rotation in a pseudomagnetic nuclear field. This effect is kinematically analogous to the effect of light polarization plane rotation in matter placed in a magnetic field.

However, it is known [1–4] that light demonstrates another interesting phenomenon: the birefringence effect in an optically anisotropic medium. This implies that as light passes through such a medium, its linear polarization (which is a vector) is converted into circular one, and then the circular polarization (which is a pseudovector) is converted into linear one.

The birefringence effect is accompanied by linear dichroism: the absorption coefficient of light with linear polarization parallel to the optic axis of matter differs from that of light with linear polarization orthogonal to the optic axis of matter.

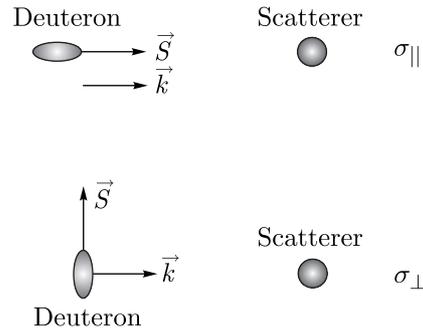


Figure 10. Explanation of deuteron birefringence effect.

It is natural to ask whether similar birefringence and dichroism effects exist for particles other than the photon.

The answer appears to be affirmative [89, 109]. According to [89, 109], a quasi-optical phenomenon of birefringence is possible for particles with spin $S \geq 1$ which move in a nonpolarized homogeneous isotropic medium.

The appearance of two refraction indices of deuteron can be easily explained.

As the ground state of deuteron is non-spherical, then it has different scattering cross-sections depending on the angle between spin and momentum of scattering deuteron.

$$\text{Im}f_{\parallel}(0) = \frac{k}{4\pi}\sigma_{\parallel} \neq \text{Im}f_{\perp}(0) = \frac{k}{4\pi}\sigma_{\perp}$$

According to the dispersion relations

$$\text{Re} f(0) \sim \Phi(\text{Im} f(0))$$

and as a result

$$\text{Re} f_{\perp}(0) \neq \text{Re} f_{\parallel}(0).$$

In this case, the vector polarization of a particle is converted into tensor polarization and vice versa.

This phenomenon also exhibits spin dichroism, which results in the fact that an initially nonpolarized deuteron beam acquires tensor polarization after passing through a nonpolarized homogeneous isotropic medium.

For particles, the birefringence effect is due to intrinsic anisotropy of particles with spin $S \geq 1$. Currently, the spin dichroism-related effect of the acquisition of tensor polarization by a nonpolarized deuteron beam which has passed through a nonpolarized target is experimentally revealed for deuterons with energies of 10–20 MeV [110–112] and deuterons with the momentum of 5 GeV/s transmitted through a carbon target [113–115]. The measured tensor polarization for deuterons with the momentum of 5 GeV/s transmitted through a carbon target as published in [115] is shown in Fig. 11.

It is worth mentioning here that for neutrons (protons) and other particles of spin $S = \frac{1}{2}$, the phenomenon similar to birefringence of light is impossible. A particle with spin $S = \frac{1}{2}$ possesses only one polarization characteristic $\sim \langle \vec{S} \rangle$ – the polarization vector which is a pseudovector. No other vector and tensor quantities may be constructed from the operators of spin $\vec{S} = \frac{1}{2}\vec{\sigma}$ alone: all the products of Pauli matrices of the type $\sigma_i\sigma_n$, etc., are finally reduced to the Pauli matrix again [6].

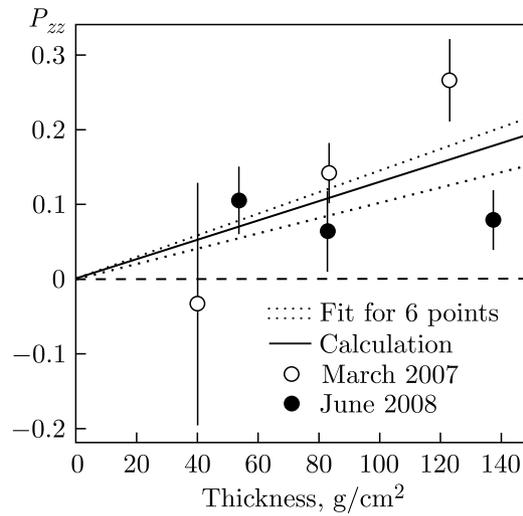


Figure 11. A tensor polarization of deuterons as a function of the thickness of the carbon target obtained in [115] (Fig. 11 therein).

The situation is different for particles with spin $S \geq 1$. In this case, the polarization state of particles is described by both the polarization vector and the polarization tensors. In particular, for a particle with spin $S = 1$ (e.g., deuteron), there are two polarization characteristics: the polarization vector $\sim \langle \vec{S} \rangle$ and the quadrupolarization tensor (tensor of rank two) $\sim S_i S_k$ (i, k correspond to the axes of the Cartesian coordinate system x, y, z) [6].

It should be pointed out here that there is a difference between the polarization state of a photon (zero rest mass) and that of a particle with a nonzero rest mass. For a photon, the polarization state with a magnetic quantum number $M = 0$ (the quantization axis is directed along the particle momentum) is missing. At the same time, such a state exists for a particle with mass.

Let us proceed to consideration of the refraction of particles with spin $S \geq 1$.

We know already (see (1)) that the refractive index of particles with spin S can be written as follows

$$\hat{N} = 1 + \frac{2\pi\rho}{k^2} \hat{f}(0),$$

where $\hat{f}(0) = \text{Tr} \hat{\rho}_J \hat{F}(0)$; $\hat{\rho}_J$ is the spin density matrix of the scatterer; $\hat{F}(0)$ is the operator amplitude of forward scattering that acts in the spin space of the particle and the scatterer with spin J .

If the particle wave function at the entrance of the target is ψ_0 , then after passing the path length z , it will be $\psi = \exp[ik\hat{N}z]\psi_0$.

The explicit form of the amplitude $\hat{f}(0)$ for particles with spin 1/2 was obtained above. In the present case, we have at our disposal three parameters for forward scattering: \vec{S} , \vec{J} , and $\vec{n} = \vec{k}/k$; \vec{k} is the particle wave vector.

It is known [6] that the spin matrix of dimensionality $(2S + 1)(2S + 1)$ can be expanded in terms of a complete set of $(2S + 1)^2$ matrices, in particular, in terms of a set of polarization operators $\hat{T}_{LM}(S)$, where $0 \ll L \ll 2S$, $-L \ll M \ll L$. The polarization operator is an irreducible tensor of rank L . The maximal rank of \hat{T}_{LM} is $2S$. The same matrix can be expanded in terms of a set of Cartesian tensors of maximal rank $2S$, i.e., in terms of a set of products $S_i S_k S_l \dots S_m$ with the maximal number of factors in this product equal to $2S$. These Cartesian tensors are reducible and may be represented as a sum of irreducible tensors. The

matrix \hat{F} operating in the spin space of \vec{S} and \vec{J} can be expanded into a set of various products of S_i and J_i in the same manner.

The most general form of such expansion allowing for the fact that \hat{F} should be scalar with respect to rotations is as follows [23, 37, 89, 109]

$$\begin{aligned} \hat{F} = & A + A_1 S_i J_i + A_2 S_i J_k n_i n_k + A_3 J_i J_k n_i n_k \\ & + A_4 S_i S_k n_i n_k + A_5 S_i S_k J_i J_k + A_6 S_i S_k n_i n_k J_l J_m n_l n_m + \dots \\ & \dots + B S_i n_i + B_1 S_i I_m e_{iml} n_l + B_2 S_i n_i S_l J_l + B_3 S_i S_l n_i n_l J_m n_m \\ & + B_4 J_i n_i + B_5 S_i J_m e_{iml} n_l S_p n_p + \dots, \end{aligned} \quad (174)$$

where the three dots stand for the terms containing the products of S_i and J_i up to $2S$ and $2J$, the terms containing A and d are caused by strong P-, T-even interactions, those with B , B_2 , B_3 , B_4 — by P-odd T-even, B_1 — by P-odd T-odd and B_5 corresponds to P-even T-odd interactions, \vec{S} is the spin of the particle incident on the target.

Upon averaging \hat{F} using the spin density matrix of the target nuclei, we find the explicit form of a coherent elastic zero-angle scattering amplitude, and hence the refractive index and the particle wave function in the target. According to (174), for particles with spin $S > 1/2$, there appear additional terms involving spin operators in the second and higher powers.

Let us find out what these terms lead to. We shall first pay attention to the fact that even in the case of a nonpolarized target, the amplitude $\hat{f}(0)$ depends on the spin operator of the incident particle and, when the quantization axis z is directed along \vec{n} , can be written in the form

$$\hat{f}(0) = d + d_1 S_z^2 + d_2 S_z^4 + \dots + d_s S_z^{2s}. \quad (175)$$

We consider a specific case of strong interactions, invariant with respect to time and space reflections; for this reason, the terms containing the odd powers of S are dropped. According to (1) and (175), the refractive index is

$$\hat{N} = 1 + \frac{2\pi\rho}{k^2} (d + d_1 S_z^2 + d_2 S_z^4 + \dots + d_s S_z^{2s}). \quad (176)$$

(176) yields an important conclusion that the refractive index of a particle with spin $S > 1/2$ depends on the spin orientation with respect to the momentum direction. Write m for a magnetic quantum number, then the refractive index of a particle in the state which is the eigenstate of the operator S_z of the spin projection on the z -axis is

$$N(m) = 1 + \frac{2\pi\rho}{k^2} (d + d_1 m^2 + d_2 m^4 + \dots + d_s m^{2s}). \quad (177)$$

According to (177), the states of a particle with quantum numbers m and $(-m)$ have the same refractive indices. For a particle with spin 1 (for example, a J/ψ -particle, deuteron) and for a particle with spin $3/2$ (for example, ^{21}Ne nucleus, Ω^- -hyperon)

$$N(m) = 1 + \frac{2\pi\rho}{k^2} (d + d_1 m^2). \quad (178)$$

As can be seen, $\text{Re}N(\pm 1) \neq \text{Re}N(0)$; $\text{Im}N(\pm 1) \neq \text{Im}N(0)$; $\text{Re}N(\pm 3/2) \neq \text{Re}N(\pm 1/2)$; $\text{Im}N(\pm 3/2) \neq \text{Im}N(\pm 1/2)$.

From this follows that for particles with spin $S > 1/2$, even a nonpolarized target causes spin dichroism: due to different absorption, the initially nonpolarized beam passing through matter acquires polarization, or more precisely, alignment [89, 109].

In view of the above analysis from (176)–(178) follows that in a medium, a moving particle with spin $S \geq 1$ has a potential energy:

$$\hat{U} = -\frac{2\pi\hbar^2\rho}{M}(d + d_1S_z^2 + d_2S_z^4 + \dots),$$

$$U(m) = -\frac{2\pi\hbar^2\rho}{M}(d + d_1m^2 + d_2m^4 + \dots).$$

The energy of interaction \hat{U} between the particle and matter is similar to that between the atom of spin $S \geq 1$ and the electric field. As a result, in the medium, the spin levels of the particle split in a way similar to Stark splitting of atomic levels in the electric field. Hence, we may say that a particle of spin $S \geq 1$, moving in matter, experiences the influence of a certain pseudoelectric field.

Since we have obtained the explicit spin structure of the refractive index, then we know the wave function ψ , and for every particular case we can find all spin characteristics of the beam in a target at depth z .

6.1. Rotation and Oscillation of Deuteron Spin in Nonpolarized Matter and Spin Dichroism (Birefringence Phenomenon)

We shall further dwell on the passage of deuterons through matter.

According to (178), the refractive indices for the states with $m = +1$ and $m = -1$ are the same, while those for the states with $m = \pm 1$ and $m = 0$ are different ($\text{Re}N(\pm 1) \neq \text{Re}N(0)$ and $\text{Im}N(\pm 1) \neq \text{Im}N(0)$).

This can be explained as follows (see Fig. 12, 13): the shape of a deuteron in the ground state is non-spherical.

Therefore, the scattering cross section $\sigma_{\pm 1}$ for a deuteron with $m = \pm 1$ (deuteron spin is parallel (antiparallel) to its momentum \vec{k}) differs from the scattering cross section σ_0 for a deuteron with $m = 0$:

$$\sigma_{\pm 1} \neq \sigma_0 \Rightarrow \text{Im}f_{\pm 1}(0) = \frac{k}{4\pi}\sigma_{\pm 1} \neq \text{Im}f_0(0) = \frac{k}{4\pi}\sigma_0. \tag{179}$$

According to the dispersion relation, $\text{Re}f(0) \sim \Phi(\text{Im}f(0))$, hence $\text{Re}f_0(0) \neq \text{Re}f_{\pm 1}(0)$

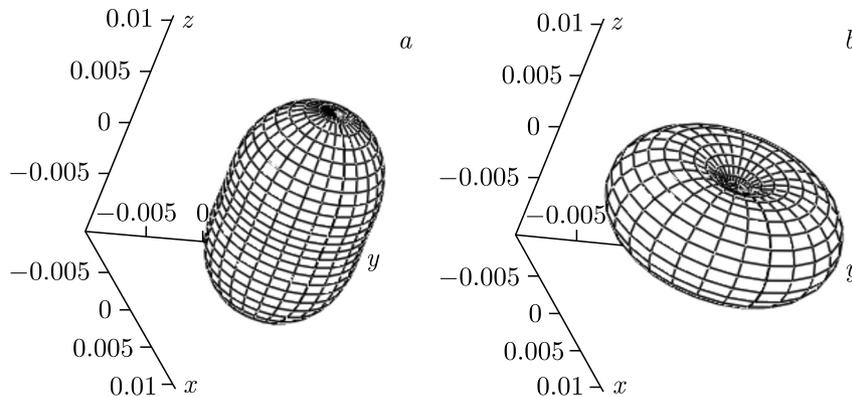


Figure 12. Squared module for deuteron ground state wave functions for the distance of 1.8 fm between its nucleons in the states a) $m = \pm 1$; b) $m = 0$.

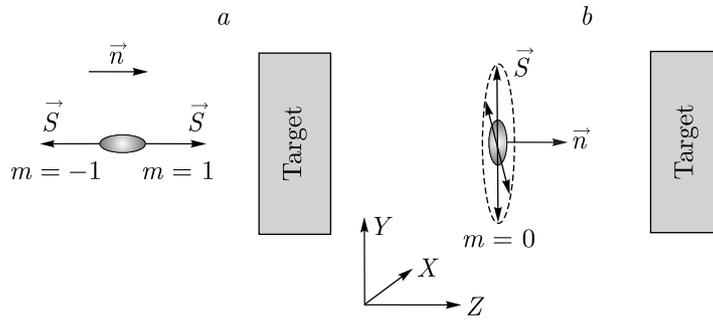


Figure 13. Two possible orientation of vectors \vec{S} and $\vec{n} = \frac{\vec{k}}{k}$: a) $m = \pm 1$; b) $m = 0$.

From the above follows that deuteron spin dichroism appears even when a deuteron passes through an nonpolarized target: owing to the fact that absorption depends on the orientation of the deuteron spin, the initially nonpolarized beam acquires alignment.

Let us consider the deuteron spin state in a target. The spin state of the deuteron is described by its vector and tensor polarizations $\vec{p} = \langle \vec{S} \rangle$ and $p_{ik} = \langle Q_{ik} \rangle$, respectively. As the deuteron moves in matter, its vector and tensor polarizations change. To calculate \vec{p} and p_{ik} , one needs to know the explicit form of the deuteron spin wave function ψ .

The wave function of the deuteron that has passed the distance z inside the target is:

$$\psi(z) = \exp\left(ik\hat{N}z\right)\psi_0, \tag{180}$$

where ψ_0 is the wave function of the deuteron before entering the target. The wave function ψ can be expressed as a superposition of the basic spin functions χ_m , which are the eigenfunctions of the operators \hat{S}^2 and \hat{S}_z ($\hat{S}_z\chi_m = m\chi_m$):

$$\psi = \sum_{m=\pm 1,0} a^m \chi_m. \tag{181}$$

Therefore

$$\psi = \begin{pmatrix} a^1 \\ a^0 \\ a^{-1} \end{pmatrix} = \begin{pmatrix} ae^{i\delta_1} e^{ikN_1z} \\ be^{i\delta_0} e^{ikN_0z} \\ ce^{i\delta_{-1}} e^{ikN_{-1}z} \end{pmatrix} = \begin{pmatrix} ae^{i\delta_1} e^{ikN_1z} \\ be^{i\delta_0} e^{ikN_0z} \\ ce^{i\delta_{-1}} e^{ikN_{-1}z} \end{pmatrix}, \tag{182}$$

according to the above, $N_1 = N_{-1}$.

Suppose that the plane (yz) coincides with the plane formed by the initial vector polarization $\vec{p}_0 \neq 0$ and the momentum \vec{k} of the deuteron. In this case

$$\delta_1 - \delta_0 = \delta_0 - \delta_{-1} = \frac{\pi}{2},$$

and the components of the polarization vector at $z = 0$ are $p_x = 0, p_y \neq 0$, and $p_z \neq 0$.

The components of the vector polarization

$$\vec{p} = \langle \vec{S} \rangle = \frac{\langle \Psi | \vec{S} | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

inside the target are:

$$\begin{aligned}
 p_x &= \frac{\sqrt{2}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z}b(a-c)\sin\left(\frac{2\pi\rho}{k}\text{Re}d_1z\right)}{\langle\Psi|\Psi\rangle}, \\
 p_y &= \frac{\sqrt{2}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z}b(a+c)\cos\left(\frac{2\pi\rho}{k}\text{Re}d_1z\right)}{\langle\Psi|\Psi\rangle}, \\
 p_z &= \frac{e^{\rho\sigma_1z}(a^2-c^2)}{\langle\Psi|\Psi\rangle}.
 \end{aligned} \tag{183}$$

Similarly, the components of the tensor polarization

$$\hat{Q}_{ij} = \frac{3}{2} \left(\hat{S}_i\hat{S}_j + \hat{S}_j\hat{S}_i - \frac{4}{3}\delta_{ij} \right)$$

are expressed as:

$$\begin{aligned}
 p_{xx} &= \frac{-\frac{1}{2}(a^2+c^2)e^{-\rho\sigma_1z} + b^2e^{-\rho\sigma_0z} - 3ace^{-\rho\sigma_1z}}{\langle\Psi|\Psi\rangle}, \\
 p_{yy} &= \frac{-\frac{1}{2}(a^2+c^2)e^{-\rho\sigma_1z} + b^2e^{-\rho\sigma_0z} + 3ace^{-\rho\sigma_1z}}{\langle\Psi|\Psi\rangle}, \\
 p_{zz} &= \frac{(a^2+c^2)e^{-\rho\sigma_1z} - 2b^2e^{-\rho\sigma_0z}}{\langle\Psi|\Psi\rangle}, \\
 p_{xy} &= 0, \\
 p_{xz} &= \frac{\frac{3}{\sqrt{2}}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z}b(a+c)\sin\left(\frac{2\pi\rho}{k}\text{Re}d_1z\right)}{\langle\Psi|\Psi\rangle}, \\
 p_{yz} &= \frac{\frac{3}{\sqrt{2}}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z}b(a-c)\cos\left(\frac{2\pi\rho}{k}\text{Re}d_1z\right)}{\langle\Psi|\Psi\rangle}, \\
 p_{xx} + p_{yy} + p_{zz} &= 0,
 \end{aligned} \tag{184}$$

where

$$\begin{aligned}
 \langle\Psi|\Psi\rangle &= (a^2+c^2)e^{-\rho\sigma_1z} + b^2e^{-\rho\sigma_0z}, \\
 \sigma_0 &= \frac{4\pi}{k}\text{Im}f_0, \quad \sigma_1 = \frac{4\pi}{k}\text{Im}f_1, \\
 f_0 &= d, \quad f_1 = d + d_1.
 \end{aligned} \tag{185}$$

According to (183), (184), spin rotation and oscillation occur when the angle between the polarization vector \vec{p} and the momentum \vec{k} of the particle differs from $\pi/2$. The magnitude of the effect is determined by the phase

$$\varphi = \frac{2\pi\rho}{k}\text{Re}d_1z. \tag{186}$$

For example, let $\text{Re}d_1 > 0$. If the angle between the polarization vector and momentum is acute, then the spin rotates anticlockwise about the momentum direction, whereas the obtuse angle between the polarization vector and the momentum gives rise to a clockwise spin rotation.

When the polarization vector and momentum are perpendicular (transversely polarized particle), the components of the vector polarization at $z = 0$ are: $p_x = 0$, $p_y \neq 0$, and $p_z = 0$. In this case $a = c$ and the dependence of the vector polarization on z can be expressed as:

$$\begin{aligned}
 p_x &= 0, \\
 p_y &= \frac{\sqrt{2}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z}2ba \cos\left(\frac{2\pi\rho}{k}\text{Re}d_1z\right)}{\langle\Psi|\Psi\rangle}, \\
 p_z &= 0, \\
 p_{xx} &= \frac{-4a^2e^{-\rho\sigma_1z} + b^2e^{-\rho\sigma_0z}}{\langle\Psi|\Psi\rangle}, \\
 p_{yy} &= \frac{2a^2e^{-\rho\sigma_1z} + b^2e^{-\rho\sigma_0z}}{\langle\Psi|\Psi\rangle}, \\
 p_{zz} &= \frac{2a^2e^{-\rho\sigma_1z} - 2b^2e^{-\rho\sigma_0z}}{\langle\Psi|\Psi\rangle}, \\
 p_{xz} &= \frac{\frac{3}{\sqrt{2}}e^{-\frac{1}{2}\rho(\sigma_0+\sigma_1)z}2ab \sin\left(\frac{2\pi\rho}{k}\text{Re}d_1z\right)}{\langle\Psi|\Psi\rangle},
 \end{aligned} \tag{187}$$

$$\begin{aligned}
 p_{yz} &= 0, \\
 p_{xx} + p_{yy} + p_{zz} &= 0.
 \end{aligned} \tag{188}$$

According to (188), no rotation occurs in this case; the vector and tensor polarization oscillate when a transversely polarized deuteron passes through matter.

Oscillations of $\langle S_x \rangle$ and Q_{yz} are opposite in phase.

Thus, if the difference $\delta_1 - \delta_0$ is zero when the particle enters the target, and consequently $\langle S_x \rangle = 1$ and $Q_{yz} = 0$, then after it has passed the path length $l_{1/4} = k(4\rho\text{Re}d_1)^{-1}$, the polarization vector vanishes while Q_{yz} attains its maximum value. With further increasing path length, the polarization vector changes sign, and Q_{yz} starts diminishing so that after this quarter of the period is over, just $\langle S_x \rangle$ attains the maximum value, while Q_{yz} vanishes, and so on. After the particle has passed four quarters of the period, the situation becomes the same as it was in the beginning when the particle entered the target. Thus, the transitions between vector and tensor polarizations of the particle appear. This effect is quite analogous to a well-known optical phenomenon of birefringence in Iceland spar. For example, after passing a quarter-wave plate, light with right-hand circular polarization becomes linearly polarized. With increasing thickness, the degree of linear polarization decreases, and a photon with left circular polarization appears. After passing this quarter of the period, the left circular polarization of the photon attains its maximum value, while its linear polarization vanishes.

Deuteron birefringence effect in matter is kinematically analogous to the oscillations of atomic spin in the electric field, which arise due to the quadratic Stark splitting of atomic levels. As mentioned above, this enables us to state that the birefringence phenomenon is caused by a pseudoelectric field acting on a particle in a medium.

6.2. The Effect of Tensor Polarization Emerging in Nonpolarized Beams Moving in Nonpolarized Matter

The effect of birefringence of particles, in particular, the appearance of tensor polarization of the initially nonpolarized beam in a real experiment, can be most clearly described using the spin density matrix [37, 111].

For deuterons (spin $S = 1$), the spin density matrix for the beam before the target can be written as follows:

$$\hat{\rho}_0 = \frac{1}{3}\hat{I} + \frac{1}{2}\vec{P}_0\hat{S} + \frac{1}{9}P_{ik}^{(0)}\hat{Q}_{ik}, \quad (189)$$

where \hat{I} is the identity (unit) matrix, \vec{P}_0 is the polarization vector of the beam, $P_{ik}^{(0)}$ is the polarization vector of the beam incident on the target. Using (180), one can express the density matrix of the deuteron beam in the target as:

$$\hat{\rho} = e^{ik\hat{N}z}\hat{\rho}_0e^{-ik\hat{N}^*z}. \quad (190)$$

As a result, we have

$$\vec{p} = \langle \vec{S} \rangle = \frac{\text{Tr}(\hat{\rho}\hat{S})}{\text{Tr}(\hat{\rho})}, \quad p_{ik} = \langle Q_{ik} \rangle = \frac{\text{Tr}(\hat{\rho}\hat{Q}_{ik})}{\text{Tr}(\hat{\rho})}, \quad (191)$$

where $i, k = x, y, z$.

In the case of thin targets, the vector and tensor polarization of the deuteron inside the target can be expressed using the first-order approximation for $e^{ik(\hat{N}-1)z} \approx 1 + ik(\hat{N}-1)z$ as follows:

$$\begin{aligned} p_x &= \frac{[1 - \frac{1}{2}\rho z(\sigma_0 + \sigma_1)]p_{x,0} + \frac{4}{3}\frac{\pi\rho z}{k}\text{Red}_1p_{zy,0}}{\text{Tr}\hat{\rho}\hat{I}}, \\ p_y &= \frac{[1 - \frac{1}{2}\rho z(\sigma_0 + \sigma_1)]p_{y,0} - \frac{4}{3}\frac{\pi\rho z}{k}\text{Red}_1p_{zx,0}}{\text{Tr}\hat{\rho}\hat{I}}, \\ p_z &= \frac{(1 - \rho\sigma_1z)p_{z,0}}{\text{Tr}\hat{\rho}\hat{I}}, \\ p_{xx} &= \frac{(1 - \rho\sigma_1z)p_{xx,0} + \frac{1}{3}\rho z(\sigma_1 - \sigma_0) - \frac{1}{3}\rho z(\sigma_1 - \sigma_0)p_{zz,0}}{\text{Tr}\hat{\rho}\hat{I}}, \\ p_{yy} &= \frac{(1 - \rho\sigma_1z)p_{yy,0} + \frac{1}{3}\rho z(\sigma_1 - \sigma_0) - \frac{1}{3}\rho z(\sigma_1 - \sigma_0)p_{zz,0}}{\text{Tr}\hat{\rho}\hat{I}}, \\ p_{zz} &= \frac{[1 - \frac{1}{3}\rho z(2\sigma_0 + \sigma_1)]p_{zz,0} - \frac{2}{3}\rho z(\sigma_1 - \sigma_0)}{\text{Tr}\hat{\rho}\hat{I}}, \\ p_{xy} &= \frac{(1 - \rho\sigma_1z)p_{xy,0}}{\text{Tr}\hat{\rho}\hat{I}}, \\ p_{xz} &= \frac{[1 - \frac{1}{2}\rho z(\sigma_0 + \sigma_1)]p_{xz,0} + 3\frac{\pi\rho z}{k}\text{Red}_1p_{y,0}}{\text{Tr}\hat{\rho}\hat{I}}, \\ p_{yz} &= \frac{[1 - \frac{1}{2}\rho z(\sigma_0 + \sigma_1)]p_{yz,0} - 3\frac{\pi\rho z}{k}\text{Red}_1p_{x,0}}{\text{Tr}\hat{\rho}\hat{I}}, \end{aligned} \quad (192)$$

where

$$\text{Tr} \hat{\rho} \hat{I} = 1 - \frac{\rho z}{3} (2\sigma_1 + \sigma_0) - \frac{\rho z}{3} (\sigma_1 - \sigma_0) p_{zz,0}.$$

If the beam is initially nonpolarized ($p_{x,0} = p_{y,0} = p_{z,0} = p_{xx,0} = p_{yy,0} = p_{zz,0} = p_{xy,0} = p_{xz,0} = p_{yz,0} = 0$), then after passing through the nonpolarized target of thickness z , the deuteron beam acquires tensor polarization:

$$\begin{aligned} p_{zz} &\approx -\frac{2}{3} \rho z (\sigma_1 - \sigma_0) = -\frac{2}{3} \rho \sigma z \frac{\Delta\sigma}{\sigma}, \\ p_{xx} = p_{yy} &\approx \frac{1}{3} \rho \sigma z \frac{\Delta\sigma}{\sigma}. \end{aligned} \quad (193)$$

Vector polarization remains equal to zero.

The expression for tensor polarization (193) may also be obtained from another viewpoint.

Let a deuteron beam in spin state with $m = 1$ pass through a target. The beam intensity changes as $I_1(z) = I_1^0 e^{-\sigma_1 \rho z}$, where I_1^0 is the beam intensity before entering the target. Similarly, for states $m = -1$ and $m = 0$, the intensity changes as $I_{-1}(z) = I_{-1}^0 e^{-\sigma_{-1} \rho z}$ and $I_0(z) = I_0^0 e^{-\sigma_0 \rho z}$, where I_{-1}^0 and I_0^0 are the beam intensities before entering the target, respectively.

Let us consider the transmission of a nonpolarized deuteron beam through a nonpolarized target.

The nonpolarized deuteron beam can be described as a composition of three polarized beams with equal intensities $I = I_1^0 + I_{-1}^0 + I_0^0$, $I_{\pm 1}^0 = I_0^0 = I/3$.

In a real experiment, $\sigma_{\pm 1,0} \rho z \ll 1$ and the change in the intensity of each beam can be expressed as $I_{\pm 1}(z) = I_{\pm 1}^0 (1 - \sigma_{\pm 1} \rho z)$ and $I_0(z) = I_0^0 (1 - \sigma_0 \rho z)$.

According to [116], the tensor polarization of the beam can be expressed as

$$p_{zz} = \frac{I_{-1} + I_1 - 2I_0}{I_{-1} + I_1 + I_0}.$$

The tensor polarization of the initially nonpolarized deuteron beam transmitting through the target of thickness L , which arises from deuteron spin dichroism, reads as follows:

$$\begin{aligned} p_{zz}(L) &= \frac{I_{-1}(L) + I_1(L) - 2I_0(L)}{I_{-1}(L) + I_1(L) + I_0(L)} \\ &\approx \frac{2N_a L (\sigma_0 - \sigma_{\pm 1})}{3M_r} = -\frac{8\pi N_a L \text{Im}(d_1)}{3kM_r}, \end{aligned} \quad (194)$$

where N_a is the Avogadro number, L is the target thickness in g/cm^2 , M_r is the molar mass of the target matter.

Note that a deuteron passing through a target loses energy by ionization of matter, then, taking into account the energy change, we can write the tensor polarization as

$$\begin{aligned} p_{zz}(L) &= \frac{2N_a}{3M_r} \int_0^L (\sigma_0(E(L')) - \sigma_{\pm 1}(E(L'))) dL' \\ &= -\frac{8\pi N_a}{3M_r} \int_0^L \frac{\text{Im}(d_1(E(L')))}{k(L')} dL'. \end{aligned} \quad (195)$$

According to (195), the imaginary part of the spin-dependent forward scattering amplitude can be measured directly in a transmission experiment by means of deuteron beam tensor polarization, which arises due to deuteron spin dichroism.

Thus, theoretical studies of the deuteron beam transmission through the nonpolarized target predict the appearance of tensor polarization in a transmitted beam due to deuteron spin dichroism.

6.3. The Amplitude of Zero-Angle Elastic Scattering of a Deuteron by a Nucleus

Let us discuss the possible magnitude of the deuteron birefringence effect in detail. According to (183), (184), (187), (188), the birefringence effect depends on the amplitudes of zero-angle elastic coherent scattering of a deuteron by a nucleus $f(m = \pm 1)$ and $f(m = 0)$.

In order to find the amplitude $f(0)$, one should start with considering the Hamiltonian H describing the interaction of the deuteron with the nucleus [89, 109].

The Hamiltonian H can be written as

$$H = H_D(\vec{r}_p, \vec{r}_n) + H_N(\{\xi_i\}) + V_{DN}(\vec{r}_p, \vec{r}_n, \{\xi_i\}), \quad (196)$$

where H_D is the deuteron Hamiltonian; H_N is the nuclear Hamiltonian; V_{DN} stands for the energy of deuteron–nucleus nuclear and Coulomb interaction; r_p and r_n are the coordinates of the proton and the neutron composing the deuteron, $\{\xi_i\}$ is the set of coordinates of the nucleons.

Having introduced the deuteron center-of-mass coordinate \vec{R} and the relative distance between the proton and the neutron in the deuteron $\vec{r} = \vec{r}_p - \vec{r}_n$, we recast (196) as

$$H = -\frac{\hbar^2}{2m_D} \Delta(\vec{R}) + H_D(\vec{r}) + H_N(\{\xi_i\}) + V_{DN}^N(\vec{R}, \vec{r}, \{\xi_i\}) + V_{DN}^C(\vec{R}, \vec{r}, \{\xi_i\}), \quad (197)$$

where $H_D(\vec{r})$ is the Hamiltonian describing the internal state of the deuteron, m_D is the deuteron mass.

In view of (197), the deuteron–nucleus scattering is determined by two interactions: nuclear and Coulomb. In this section, we shall content ourselves with finding the amplitude of forward elastic scattering of a deuteron with energy of hundreds of megaelectronvolts by a light nucleus due to nuclear interaction (the term V_{DN}^C in (197) will be ignored). At lower energies, taking account of the Coulomb interaction is essential [117].

In further consideration, we shall pay attention to the fact that for deuterons, for example, with energy of several tens of megaelectronvolts, appreciably exceeding the binding energy of deuterons ε_d , the time of nuclear deuteron–nucleus interaction is $\tau^N \simeq 5 \cdot 10^{-22}$ s, whereas the characteristic period of oscillation of nucleus in the deuteron is $\tau \simeq 2\pi\hbar/\varepsilon_d \simeq 2 \cdot 10^{-21}$ s. So we can apply the impulse approximation [15]. In this approximation, we can neglect the binding energy of nucleons in the deuteron, i.e., neglect $H_D(\vec{r})$ in (197). As a result,

$$H = -\frac{\hbar^2}{2m_D} \Delta(\vec{R}) + H_N(\{\xi_i\}) + V_{DN}^N(\vec{R}, \vec{r}, \{\xi_i\}). \quad (198)$$

As can be seen, in the impulse approximation, the problem of determining the scattering amplitude reduces to the problem of scattering by a nucleus of a structureless particle having the same mass as the deuteron. In this case, the coordinate \vec{r} is a parameter. Therefore, the relations obtained for the cross section and the forward scattering amplitude should be averaged over the stated parameter. To estimate the magnitude of the effect, we shall also neglect the spin-dependence of internucleonic interaction. This enables using eikonal approximation for analyzing the magnitude of the amplitude for fast deuterons [118].

In this approximation, the amplitude of coherent zero-angle scattering can be written as follows:

$$f(0) = \frac{k}{2\pi i} \int \left(e^{i\chi_D(\vec{b}, \vec{r})} - 1 \right) d^2b |\varphi(\vec{r})|^2 d^3r, \tag{199}$$

where k is the deuteron wave number, \vec{b} is the impact parameter, $\varphi(\vec{r})$ is the wave function of the deuteron ground state. The phase shift due to the deuteron scattering by carbon is

$$\chi_D = -\frac{1}{\hbar v} \int_{-\infty}^{+\infty} V_{DN}(\vec{b}, z', \vec{r}_\perp) dz', \tag{200}$$

\vec{r}_\perp is the component of \vec{r} , which is perpendicular to the momentum of incident deuteron, v is the deuteron speed. The phase shift $\chi_D = \chi_1 + \chi_2$, where χ_1 and χ_2 are the phase shifts caused by proton-nucleus and neutron-nucleus interactions, respectively.

For the deuteron, the probability $|\varphi(\vec{r})|^2$ differs for different spin states. Thus, for states with magnetic quantum number $m = \pm 1$, the probability is $|\varphi_{\pm 1}(\vec{r})|^2$, whereas for $m = 0$, it is $|\varphi_0(\vec{r})|^2$.

Owing to the additivity of phase shifts, equation (199) can be rewritten as

$$f(0) = \frac{k}{\pi} \int \left\{ t_1\left(\vec{b} - \frac{\vec{r}_\perp}{2}\right) + t_2\left(\vec{b} + \frac{\vec{r}_\perp}{2}\right) + 2it_1\left(\vec{b} - \frac{\vec{r}_\perp}{2}\right) t_2\left(\vec{b} + \frac{\vec{r}_\perp}{2}\right) \right\} \times |\varphi(\vec{r})|^2 d^2b d^3r, \tag{201}$$

where

$$t_{1(2)} = \frac{e^{i\chi_{1(2)}} - 1}{2i}.$$

From (201) follows

$$f(0) = f_1(0) + f_2(0) + \frac{2ik}{\pi} \int t_1\left(\vec{b} - \frac{\vec{r}_\perp}{2}\right) t_2\left(\vec{b} + \frac{\vec{r}_\perp}{2}\right) |\varphi(\vec{r}_\perp, z)|^2 d^2b d^2r_\perp dz, \tag{202}$$

where

$$f_{1(2)}(0) = \frac{k}{\pi} \int t_{1(2)}(\vec{\xi}) d^2\xi = \frac{m_D}{m_{p(n)}} f_{p(n)}(0)$$

and $f_{p(n)}(0)$ is the amplitude of the proton-(neutron)-nucleus zero-angle elastic coherent scattering. (202) can be recast as

$$f(0) = f_1(0) + f_2(0) + \frac{2ik}{\pi} \int t_1(\vec{\xi}) t_2(\vec{\eta}) \left| \varphi(\vec{\xi} - \vec{\eta}, z) \right|^2 d^2\xi d^2\eta dz. \tag{203}$$

Then from (203), we get

$$\begin{aligned} \operatorname{Re} f(0) &= \operatorname{Re} f_1(0) + \operatorname{Re} f_2(0) - \frac{2k}{\pi} \operatorname{Im} \int t_1(\vec{\xi}) t_2(\vec{\eta}) \\ &\times \left| \varphi(\vec{\xi} - \vec{\eta}, z) \right|^2 d^2\xi d^2\eta dz, \\ \operatorname{Im} f(0) &= \operatorname{Im} f_1(0) + \operatorname{Im} f_2(0) + \frac{2k}{\pi} \operatorname{Re} \int t_1(\vec{\xi}) t_2(\vec{\eta}) \\ &\times \left| \varphi(\vec{\xi} - \vec{\eta}, z) \right|^2 d^2\xi d^2\eta dz. \end{aligned} \tag{204}$$

In accordance with (183), (184), the polarization state of the deuteron in the target is determined by the difference of the amplitudes $\text{Re}f(m = \pm 1)$ and $\text{Re}f(m = 0)$, and $\text{Im}f(m = \pm 1)$ and $\text{Im}f(m = 0)$.

From (201) follows that [89, 109, 117]

$$\begin{aligned} \text{Re}d_1 &= -\frac{2k}{\pi} \text{Im} \int t_1(\vec{\xi})t_2(\vec{\eta}) \left[\varphi_{\pm 1}^+ \left(\vec{\xi} - \vec{\eta}, z \right) \varphi_{\pm 1} \left(\vec{\xi} - \vec{\eta}, z \right) \right. \\ &\quad \left. - \varphi_0^+ \left(\vec{\xi} - \vec{\eta}, z \right) \varphi_0 \left(\vec{\xi} - \vec{\eta}, z \right) \right] d^2\xi d^2\eta dz \\ \text{Im}d_1 &= \frac{2k}{\pi} \text{Re} \int t_1(\vec{\xi})t_2(\vec{\eta}) \left[\varphi_{\pm 1}^+ \left(\vec{\xi} - \vec{\eta}, z \right) \varphi_{\pm 1} \left(\vec{\xi} - \vec{\eta}, z \right) \right. \\ &\quad \left. - \varphi_0^+ \left(\vec{\xi} - \vec{\eta}, z \right) \varphi_0 \left(\vec{\xi} - \vec{\eta}, z \right) \right] d^2\xi d^2\eta dz. \end{aligned} \tag{205}$$

Note that according to (205), the spin-dependent part of the scattering amplitude d_1 is determined by the rescattering effects of colliding particles.

When the deuteron is scattered by a light nucleus, its characteristic radius is large as compared with the radius of the nucleus. For this reason, to estimate the effects, we can suppose that in integration, the functions t_1 and t_2 act on φ as a δ -function. Then

$$\begin{aligned} \text{Re}d_1 &= -\frac{4\pi}{k} \text{Im}f_1(0)f_2(0) \int_0^\infty \left[\varphi_{\pm 1}^+ (0, z) \varphi_{\pm 1} (0, z) \right. \\ &\quad \left. - \varphi_0^+ (0, z) \varphi_0 (0, z) \right] dz, \\ \text{Im}d_1 &= \frac{4\pi}{k} \text{Re}f_1(0)f_2(0) \int_0^\infty \left[\varphi_{\pm 1}^+ (0, z) \varphi_{\pm 1} (0, z) \right. \\ &\quad \left. - \varphi_0^+ (0, z) \varphi_0 (0, z) \right] dz. \end{aligned} \tag{206}$$

The magnitude of the birefringence effect is determined by the difference

$$\left[\varphi_{\pm 1}^+ (0, z) \varphi_{\pm 1} (0, z) - \varphi_0^+ (0, z) \varphi_0 (0, z) \right],$$

i.e., by the difference of distributions of nucleon density in the deuteron for different deuteron spin orientations. As noted hereinbefore, the spin dichroism effect (the effect of birefringence of particles, nuclei) is caused by the internal anisotropy of nuclei (particles) with spin $S \geq 1$. Therefore, the effect magnitude can always be evaluated as follows: it is sufficient, for example, to multiply the total cross section by the degree of asymmetry caused by the difference in areas (volumes) of spherical and ellipsoidal nuclei shapes. The structure of the wave function $\varphi_{\pm 1}$ is well known [119]:

$$\varphi_m = \frac{1}{\sqrt{4\pi}} \left\{ \frac{u(r)}{r} + \frac{1}{\sqrt{8}} \frac{W(r)}{r} \hat{S}_{12} \right\} \chi_m, \tag{207}$$

where $u(r)$ is the deuteron radial wave function corresponding to the S -wave; $W(r)$ is the radial function corresponding to the D -wave; the operator $\hat{S}_{12} = 6(\hat{S}\vec{n}_r)^2 - 2\hat{S}^2$; $\vec{n}_r = \frac{\vec{r}}{r}$; $\hat{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$, and $\vec{\sigma}_{1(2)}$ are the Pauli spin matrices describing proton (neutron) spin.

Use of (207) yields

$$\begin{aligned} \text{Re}d_1 &= -\frac{6}{k} \text{Im} \{f_1(0)f_2(0)\} G = -\frac{24}{k} \text{Im} \{f_p(0)f_n(0)\} G, \\ \text{Im}d_1 &= \frac{6}{k} \text{Re} \{f_1(0)f_2(0)\} G = \frac{24}{k} \text{Re} \{f_p(0)f_n(0)\} G, \end{aligned} \tag{208}$$

where

$$G = \int_0^\infty \left(\frac{1}{\sqrt{2}} \frac{u(r)W(r)}{r^2} - \frac{1}{4} \frac{W^2(r)}{r^2} \right) dr.$$

According to the optical theorem,

$$\text{Im } f_{p(n)}(0) = \frac{k_{p(n)}}{4\pi} \sigma_{p(n)},$$

where $\sigma_{p(n)}$ is the total scattering cross section of the proton and the neutron by carbon, respectively, and

$$k_{p(n)} = \frac{m_{p(n)}}{m_D} k \simeq \frac{1}{2} k.$$

As a result, (208) can be written as

$$\text{Re } d_1 = -\frac{3}{\pi} (\text{Re } f_p(0)\sigma_n + \text{Re } f_n(0)\sigma_p) G \quad (209)$$

$$\text{Im } d_1 = \left(\frac{24}{k} \text{Re } f_p(0) \text{Re } f_n(0) - \frac{3k}{8\pi^2} \sigma_p \sigma_n \right) G. \quad (210)$$

In view of (209), the analysis of the birefringence phenomenon in this simple approximation provides information about the relation between the real part of the amplitude of the proton–nucleus zero-angle scattering (nucleon–nucleon scattering in the case of interaction between deuterons and a hydrogen target). Expressions (209) and (210) enable making first estimates of the magnitude of the effect and demonstrating the real possibility of its experimental observation [89, 106, 117]. In particular, for phase φ , determining the spin rotation angle of the deuteron, the following value was obtained for energies up to hundreds of megavolts for a carbon target:

$$\varphi = \frac{2\pi\rho z}{k} \text{Re } d_1 \simeq 10^{-3} z,$$

z is the path length (in centimeters) traveled by the particle in the target.

The magnitude of dichroism and tensor polarization acquired by a nonpolarized beam after passing the path length z is $P_{zz} \approx 10^{-2} z$ [106].

The estimate of the magnitudes of the effect based on the eikonal approximation formulas (201) ignores the spin dependence of the nucleon–nucleon interaction. The Glauber multiple scattering theory, generalized to the case when the spin dependence of nucleon–nucleon interaction is included, enables one to take account of this dependence [120, 121]. The analysis including the stated dependence was carried out in [106]. In view of this analysis, the spin dependence of the nucleon–nucleon interaction influences the birefringence effect in the range of deuteron energies less than several GeV in such a way that the estimates carried out using a simplified approach remains valid.

6.4. First Observation of Spin Dichroism with Deuterons in a Carbon Target

In 2003, the first experimental observation of the deuteron spin dichroism (the effect of the appearance of tensor polarization in a deuteron beam transmitted through a carbon target) was carried out at the electrostatic HVEC tandem Van-de-Graaff accelerator with deuteron energy up to 20 MeV (Institut für Kernphysik of Universität zu Köln) [110–112]. These experiments revealed that in the energy interval 5–20 MeV, both the value and the sign of the deuteron tensor polarization in a carbon target change with changing deuteron energy.

In 2007, spin dichroism and the effect of appearance of tensor polarization for 5.5 GeV/s deuterons transmitted through carbon targets were observed in another experiment at the Nuclotron in JINR (Russia) [113]. Taking into account both nuclear interaction and the Coulomb interaction between the deuteron and the nucleus enables one to describe the effect of changing sign of tensor polarization with the change in the deuteron energy, which was observed in the experiments with the deuterons of energy $5 \div 20$ MeV [89, 109, 117]. However, the magnitude of the effect has not been explained yet [122].

For high energy deuterons, the nuclear interaction of a deuteron with a light nucleus gives the main contribution to the birefringence effect. Comparison of the theoretical estimation of [89, 109], which includes the consideration of nuclear interaction, with the experimental results of [113] demonstrates their qualitative agreement.

Moreover, the sign of the effect obtained in theoretical calculations coincides with that measured in the experiment.

It is necessary to add that, as follows from the experiment, the phenomenon of deuteron spin dichroism can be used for obtaining a source of tensor-polarized deuterons [110–114, 122].

Thus, theoretically predicted phenomena of spin dichroism and appearance of tensor polarization for deuterons transmitted through matter [89, 109] have been revealed in the experiment. It should be recalled that, in addition to dichroism, the effect of deuteron birefringence is accompanied by spin oscillations, rotation and conversion of vector polarization to tensor polarization and vice versa, which are still awaiting experimental observation.

6.5. Tensor polarization rotation and oscillation between vector and tensor polarizations

Let us examine the equations (192) more carefully. These equations describe the change of spin characteristics of a deuteron (nucleus with spin 1) during its motion in a target. The change of vector polarization as the particle moves through matter is described by the first three equations for polarization components p_x , p_y , and p_z . Let us consider the equations for p_x and p_y :

Hereafter, unless otherwise specified, we consider the target to be thin, so that birefringence effects are small.

$$\begin{aligned} p_x &= \frac{\left[1 - \frac{1}{2}\rho z (\sigma_0 + \sigma_1)\right] p_{x,0} + \frac{4}{3} \frac{\pi\rho z}{k} \text{Red}_1 p_{yz,0}}{\text{Tr} \hat{\rho} \hat{I}}, \\ p_y &= \frac{\left[1 - \frac{1}{2}\rho z (\sigma_0 + \sigma_1)\right] p_{y,0} - \frac{4}{3} \frac{\pi\rho z}{k} \text{Red}_1 p_{xz,0}}{\text{Tr} \hat{\rho} \hat{I}}, \end{aligned} \quad (211)$$

According to (211), while the particle moves through matter, nonzero initial tensor polarization ($p_{yz,0}$ and $p_{xz,0}$) leads to rotation of the polarization vector. Let us consider a pair of equations (211) for vector polarization components p_x and p_y . Suppose that component p_x is equal to zero when particle enters the target ($p_{x,0} = 0$). In this case, tensor polarization component p_{xz} presenting in (211) is also equal to zero ($p_{xz,0} = 0$). Polarization vector lays in plane (y, z) (see Fig. 14). Equations (211) for this case can be expressed as follows:

$$\begin{aligned} p_x &= \frac{4}{3} \frac{\pi\rho z}{k} \text{Red}_1 p_{yz,0}, \\ p_y &= \frac{\left[1 - \frac{1}{2}\rho z (\sigma_0 + \sigma_1)\right]}{\text{Tr} \hat{\rho} \hat{I}} p_{y,0}, \end{aligned} \quad (212)$$

It is clear that the appearance of nonzero component p_x means rotation of polarization vector. The angle of rotation can be evaluated as follows:

$$\vartheta \approx \frac{p_x}{p_y} = \frac{4\pi}{3} \frac{\rho z}{k} \text{Red}_1 \frac{p_{yz,0}}{p_{y,0}}.$$

In the specific case when in the deuteron wavefunction the state with $m = -1$ is absent (amplitude c in (182) is equal to zero) and the ratio $\frac{p_{yz,0}}{p_{y,0}} = \frac{3}{2}$, the angle of rotation reads as follows:

$$\vartheta \approx 2\pi\lambda_c \frac{1}{\gamma} \text{Red}_1 z.$$

Suppose the nucleus entering the target does not possess vector polarization but has tensor polarization. In this case, tensor polarization causes the appearance of vector polarization.

And vice versa, according to expressions (192) for tensor polarization:

$$\begin{aligned} p_{xz} &= \frac{\left[1 - \frac{1}{2}\rho z (\sigma_0 + \sigma_1)\right] p_{xz,0} + 3\frac{\pi\rho z}{k} \text{Red}_1 p_{y,0}}{\text{Tr}\hat{\rho}\hat{I}}, \\ p_{yz} &= \frac{\left[1 - \frac{1}{2}\rho z (\sigma_0 + \sigma_1)\right] p_{yz,0} - 3\frac{\pi\rho z}{k} \text{Red}_1 p_{x,0}}{\text{Tr}\hat{\rho}\hat{I}}. \end{aligned} \quad (213)$$

Vector polarization leads to the appearance of tensor polarization components p_{yz} and p_{xz} if the particle entering the target did not possess them. At the same time, the vector polarization component $p_{x,0}$ leads to the appearance of p_{yz} , and the component $p_{y,0}$ to the appearance of p_{xz} . If both tensor polarization components $p_{yz,0}$ and $p_{xz,0}$ (or one of the components either $p_{yz,0}$ or $p_{x,0}$) are nonzero upon entering the target, rotation also arises. Evolution of vector and tensor polarization is similar to the axis precession of a gyroscope rotating and moving over complicated trajectory. A separate paper will be dedicated to exploring this issue in detail.

From (212) it follows that the angle of rotation for polarization vector for a deuteron beam passing distance l in a thin target with density ρ (the number of nuclei per cm^3) reads:

$$\vartheta = 2\pi\rho l\lambda_c \frac{1}{\gamma} \text{Red}_1 \simeq 10^{-3} l,$$

where $\lambda_c = \frac{\hbar}{Mc}$ is the Compton wavelength.

Analysis shows that for deuterons of several GeV energies passing through a carbon target with thickness of several nuclear interaction lengths, the angle of polarization vector (polarization tensor) rotation is about $\vartheta \sim 1 \cdot 10^{-2}$ rad.

For an acute and obtuse angles between spin and momentum rotation occurs in opposite directions (see Fig. 14).

The evaluation of the spin polarization rotation effect for a deuteron beam in a carbon target is given by Table 2 for conditions similar to those for deuteron dichroism observation [115], namely: deuteron momentum 5 GeV/c and carbon target thickness $\rho l = 83 \text{ g}\cdot\text{cm}^{-2}$. In experiment [115] tensor polarization component p_{zz} was measured to be as high as $p_{zz} \approx 0.1$ that corresponded to $\Delta\sigma/\sigma \approx 0.06$. The rotation of the polarization vector is revealed by acquiring components p_x and the rotation angle is evaluated in Table 2 for a number of deuterons in a bunch N_b . Evaluations show that the time required to observe the discussed phenomena for beams of polarized deuterons at the Nuclotron-M is about 10–30 hours.

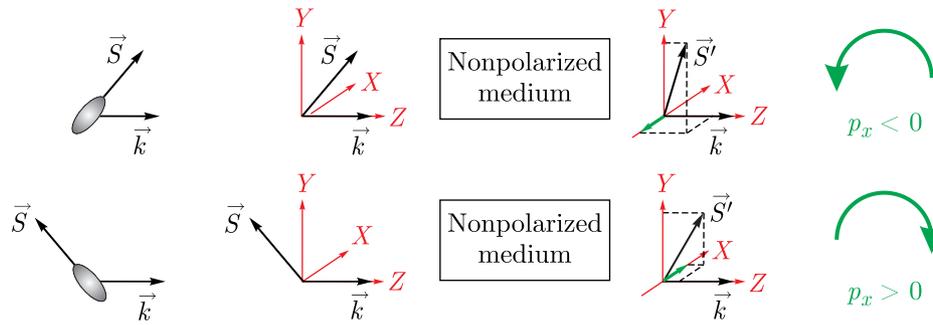


Figure 14. Rotation for acute and obtuse angles between spin and momentum occurs in opposite directions.

Table 2. Evaluation of the deuteron spin rotation effect for carbon target.

N_b	σ , b	ρl , $\text{g} \cdot \text{cm}^{-2}$	$\Delta\sigma/\sigma$	$ \frac{\text{Re}(d_1)}{\text{Im}(d_1)} $	$\vartheta \approx \frac{ p_x }{ p_y }$
10^{10}	0.6	83	0.06	0.1	$4 \cdot 10^{-3}$

6.6. Dichroism effect, rotation of spin and tensor polarization for heavy nuclei

All the phenomena considered above, which are caused by the birefringence effect (spin dichroism, spin rotation and tensor polarization, transition of vector polarization into tensor and tensor into vector), exist for all particles with spin $S \geq 1$. It is very important to note that, as mentioned hereinabove, the birefringence effect associated with particles motion through nonpolarized matter is caused by particle’s internal anisotropy. For nuclei heavier than deuteron, this anisotropy could significantly exceed that for deuterons. For example, for the ^{21}Ne nucleus, the deviation of the nuclear shape from spherical is about 30 percent. Many other nuclei possess a noticeable rate of anisotropy.

Therefore, an initially nonpolarized beam of heavy nuclei with spin $S \geq 1$ passing through a nonpolarized external target at the Nuclotron would acquire tensor polarization due to spin dichroism.

For example, a beam of ^{21}Ne nuclei, which possess spin 3/2 and large quadrupole deformation ($\beta_2=0.463$), acquires tensor polarization, which value is the higher, the longer is the nuclei path in the target. If the beam passes through the external target at the Nuclotron distance $l = 4.6 L_{\text{nuc}}$, it is attenuated 100 times ($N/N_0 = 10^{-2}$), attenuation 10^4 times corresponds to $l = 9.2 L_{\text{nuc}}$, here $L_{\text{nuc}} = \frac{1}{n\sigma}$, n is the number of scatterers in the target per cm^3 , σ is the total cross-section of Ne scattering on the target nuclei, σnl has the meaning of target thickness in nuclear lengths. According to evaluations [123, 125, 142], the tensor polarization of ^{21}Ne beam in carbon target appears as high as $p_{zz} = 0.56$ at $\sigma nl = 4.6$ and $p_{zz} = 0.85$ at $\sigma nl = 9.2$. Evaluations carried by Lebedev and Shimansky for ^{21}Ne beam passing through 20 cm thick beryllium target give similar results: tensor polarization $p_{zz} \sim 0.4$ can be obtained [124]. All the results are gathered in Table 3 and Fig. 15. In these calculations, $\Delta\sigma/\sigma$ was supposed to be $\Delta\sigma/\sigma = 0.133$ and 0.128 for beryllium and carbon targets, respectively.

Great value of tensor polarization appearing due to spin dichroism for many nuclei gave possibility to V. Lebedev and C. Shimansky to formulate an idea for the application of nuclei, which acquire tensor polarization at low energies, for further acceleration and injection into the NICA ring. Therefore, one gains the possibility to study reactions induced at collisions of tensor polarized nuclei beams.

Table 3. Evaluation of tensor polarization acquired by ^{21}Ne nuclei in carbon and beryllium targets for different beam attenuation values.

N/N_0	σnl	p_{zz} (Be)	p_{zz} (C)
10^{-2}	4.6	0.58	0.56
10^{-4}	9.2	0.87	0.85

Certainly, it is also necessary to consider experiments with heavy nuclei enabling to observe the effect of tensor polarization rotation and that of tensor polarization conversion to vector polarization.

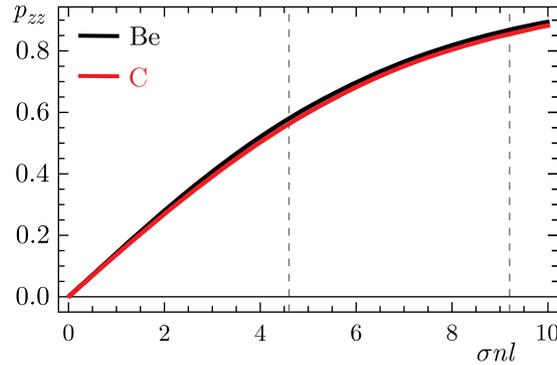


Figure 15. Dependence of the tensor polarization p_{zz} acquired by ^{21}Ne nuclei passing through carbon and beryllium targets. Vertical dashed lines in the plot mark target thickness $\sigma nl = 4.6$ and 9.2 corresponding to beam attenuation 100 and 10^4 times, respectively.

6.7. Possibilities to observe spin dichroism at the internal target of the Nuclotron-M

Let us consider deuteron beam motion in a storage ring in the presence of external magnetic fields. The spin precession of the particle, caused by the interaction of the particle magnetic moment with the external magnetic field, is described by the Bargmann–Michel–Telegdi equation [62, 127]

$$\frac{d\vec{p}}{dt} = [\vec{p} \times \vec{\Omega}_0], \quad (214)$$

where t is the time in the laboratory frame,

$$\vec{\Omega}_0 = \frac{e}{mc} \left[\left(a + \frac{1}{\gamma} \right) \vec{B} - a \frac{\gamma}{\gamma + 1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} \right], \quad (215)$$

m is the particle mass, e is its charge, \vec{p} is the polarization vector, γ is the Lorentz factor, $\vec{\beta} = \vec{v}/c$, \vec{v} is the particle velocity, $a = (g - 2)/2$, g is the gyromagnetic ratio, \vec{B} is the magnetic field at the particle location.

Thus, evolution of the deuteron spin is described by the following equation:

$$\frac{d\vec{p}}{dt} = \frac{e}{mc} \left[\vec{p} \times \left\{ \left(a + \frac{1}{\gamma} \right) \vec{B} - a \frac{\gamma}{\gamma + 1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} \right\} \right]. \quad (216)$$

However, the equation (216) alone is not sufficient to describe spin evolution in the Nuclotron with an internal target: it is necessary to supplement it with a contribution caused by the

interaction of the deuteron with the internal target. This interaction is described by effective potential energy \hat{V} , which a particle in matter possesses [37, 89, 109]:

$$\hat{V} = -\frac{2\pi\hbar^2}{M}\rho\hat{f}(0), \quad (217)$$

where $\hat{f}(0)$ is the amplitude of elastic coherent forward scattering, the explicit expression of $\hat{f}(0)$ for a particle with spin $S \geq 1$ (see (174)) was obtained in [37, 89, 109]. If weak interactions are neglected, it converts to the following form:

$$f(\hat{0}) = d + d_1 \left(\hat{S}\vec{n} \right)^2, \quad (218)$$

where \vec{n} is the unit vector in the direction of particle momentum.

The density matrix of a system “deuteron beam + target” can be expressed as [73]:

$$\hat{\rho} = \hat{\rho}_d \otimes \hat{\rho}_t, \quad (219)$$

where $\hat{\rho}_d$ is the density matrix of a deuteron beam, $\hat{\rho}_t$ density matrix of a target. The density matrix of a deuteron beam

$$\hat{\rho}_d = I(\vec{k}) \left(\frac{1}{3}\hat{1} + \frac{1}{2}\vec{p}(\vec{k})\hat{S} + \frac{1}{9}p_{ik}(\vec{k})\hat{Q}_{ik} \right), \quad (220)$$

$I(\vec{k})$ is the beam intensity, \vec{p} is the polarization vector, p_{ik} is the polarization tensor for the deuteron beam.

Equation for density matrix of a deuteron beam reads as:

$$\frac{d\hat{\rho}_d}{dt} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}_d] + \left(\frac{\partial \hat{\rho}_d}{\partial t} \right)_{\text{col}}, \quad (221)$$

where $\hat{H} = \hat{H}_0 + \hat{V}$.

The term responsible for collisions $\left(\frac{\partial \hat{\rho}_d}{\partial t} \right)_{\text{col}}$ can be obtained with the use of method described in [73] and section 5.3:

$$\left(\frac{\partial \hat{\rho}_d}{\partial t} \right)_{\text{col}} = vNSp_t \left[\frac{2\pi i}{k} \left[\hat{F}(\theta=0)\hat{\rho} - \hat{\rho}\hat{F}^+(\theta=0) \right] + \int d\Omega \hat{F}(\vec{k}')\hat{\rho}(\vec{k}')\hat{F}^+(\vec{k}') \right], \quad (222)$$

where $\vec{k}' = \vec{k} + \vec{q}$, \vec{q} is the momentum transferred from the incident particle to matter, v is the velocity of the incident particle, N is the number of atoms in cm^3 of matter, \hat{F} is the scattering amplitude, which depends on the spin operators of deuterons and nuclei (atoms) of matter, \hat{F}^+ is the operator Hermitian conjugate to operator \hat{F} . The first term in (222) describes coherent scattering of the particle by the nuclei of matter, while the second one is responsible for multiple scattering.

Let us consider the first term in (222) in more details:

$$\left(\frac{\partial \hat{\rho}_d}{\partial t} \right)_{\text{col}}^{(1)} = vN\frac{2\pi i}{k} \left[\hat{f}(0)\hat{\rho}_d - \hat{\rho}_d\hat{f}(0)^+ \right]. \quad (223)$$

Amplitude $\hat{f}(0)$ of forward scattering of a deuteron in a nonpolarized target can be expressed as:

$$\hat{f}(0) = Sp_t\hat{F}(0)\hat{\rho}_t. \quad (224)$$

In accordance with (218), the amplitude reads:

$$\hat{f}(0) = d + d_1(\hat{S}\vec{n})^2, \quad (225)$$

where $\vec{n} = \vec{k}/k$, \vec{k} is the deuteron momentum. As a result, expression (223) reads as follows:

$$\left(\frac{\partial \hat{\rho}_d}{\partial t}\right)_{\text{col}}^{(1)} = -\frac{i}{\hbar} \left(\hat{V}\hat{\rho}_d - \hat{\rho}_d\hat{V}^+\right). \quad (226)$$

where \hat{V} is defined by (217). And finally, equation (221) can be presented as follows:

$$\frac{d\hat{\rho}_d}{dt} = -\frac{i}{\hbar} [\hat{H}, \hat{\rho}_d] - \frac{i}{\hbar} \left(\hat{V}\hat{\rho}_d - \hat{\rho}_d\hat{V}^+\right) + vN Sp_t \int d\Omega F(\vec{k}') \hat{\rho}(\vec{k}') F^+(\vec{k}'). \quad (227)$$

The last term, proportional to Sp_t , describes multiple scattering and the resulting depolarization (for more details, see section 5.3 and [98–101]). Hereinafter, the target thickness enabling to neglect this term is used.

The beam intensity reads as

$$I(t) = Sp_d \hat{\rho}_d. \quad (228)$$

Therefore, the rate of intensity change is determined by scattering amplitude $\hat{f}(0)$ as follows:

$$\frac{dI}{dt} = vN \frac{2\pi i}{k} Sp_d \left[\hat{f}(0)\hat{\rho}_d - \hat{\rho}_d\hat{f}^+(0)\right]. \quad (229)$$

Substituting (220) and (225) to (229), one can get the rate of intensity change caused by the tensor polarization components as follows

$$\frac{dI}{dt} = \frac{\chi}{3} [2 + p_{ik}n_in_k] I(t) + \alpha I(t), \quad (230)$$

where parameters $\chi = -\frac{4\pi vN}{k} \text{Im}d_1 = -vN(\sigma_{\pm 1} - \sigma_0)$ and $\alpha = -\frac{4\pi vN}{k} \text{Im}d = -vN\sigma_0$ depend on total scattering cross sections $\sigma_{\pm 1}$ and σ_0 for quantum numbers $m = \pm 1$ and $m = 0$, respectively.

Vector polarization \vec{p} of a deuteron beam reads as follows:

$$\vec{p} = \frac{Sp_d \hat{\rho}_d \hat{S}}{Sp_d \hat{\rho}_d} = \frac{Sp_d \hat{\rho}_d \hat{S}}{I(t)}. \quad (231)$$

Differential equations describing vector polarization can be obtained from (231):

$$\frac{d\vec{p}}{dt} = \frac{Sp_d(d\hat{\rho}_d/dt)\hat{S}}{I(t)} - \vec{p} \frac{Sp_d(d\hat{\rho}_d/dt)}{I(t)}. \quad (232)$$

The tensor polarization components p_{ik} are defined as:

$$p_{ik} = \frac{Sp_d \hat{\rho}_d \hat{Q}_{ik}}{Sp_d \hat{\rho}_d} = \frac{Sp_d \hat{\rho}_d \hat{Q}_{ik}}{I(t)}, \quad (233)$$

where quadrupolarization operator \hat{Q}_{ik} reads: $\hat{Q}_{ik} = \frac{3}{2} \left(\hat{S}_i \hat{S}_k + \hat{S}_k \hat{S}_i - \frac{4}{3} \delta_{ik} \hat{\mathbf{I}} \right)$. Similar to the vector polarization, the following equation describes evolution of the tensor polarization

$$\frac{dp_{ik}}{dt} = \frac{Sp_d(d\hat{\rho}_d/dt)\hat{Q}_{ik}}{I(t)} - p_{ik} \frac{Sp_d(d\hat{\rho}_d/dt)}{I(t)}. \quad (234)$$

Combining equations (220) and (214), (232) and (234), along with condition $p_{xx} + p_{yy} + p_{zz} = 0$, yields a system [73, 126] describing the evolution of both vector and tensor polarization components for a deuteron:

$$\left\{ \begin{array}{l} \frac{d\vec{p}}{dt} = \frac{e}{mc} \left[\vec{p} \times \left\{ \left(a + \frac{1}{\gamma} \right) \vec{B} - a \frac{\gamma}{\gamma+1} \left(\vec{\beta} \cdot \vec{B} \right) \vec{\beta} \right\} \right] + \\ + \frac{\chi}{2} (\vec{n}(\vec{n} \cdot \vec{p}) + \vec{p}) + \\ + \frac{\eta}{3} [\vec{n} \times \vec{n}'] - \frac{2\chi}{3} \vec{p} - \frac{\chi}{3} (\vec{n} \cdot \vec{n}') \vec{p}, \\ \frac{dp_{ik}}{dt} = - (\varepsilon_{jkr} p_{ij} \Omega_r + \varepsilon_{jir} p_{kj} \Omega_r) + \\ + \chi \left\{ -\frac{1}{3} + n_i n_k + \frac{1}{3} p_{ik} - \frac{1}{2} (n'_i n_k + n_i n'_k) + \frac{1}{3} (\vec{n} \cdot \vec{n}') \delta_{ik} \right\} + \\ + \frac{3\eta}{4} ([\vec{n} \times \vec{p}]_i n_k + n_i [\vec{n} \times \vec{p}]_k) - \frac{\chi}{3} (\vec{n} \cdot \vec{n}') p_{ik}, \end{array} \right. \quad (235)$$

where $\vec{n} = \vec{k}/k$, $\eta = -\frac{4\pi N}{k} \text{Re}d_1$, $n'_i = p_{ik} n_k$, Ω_r are the components of $\vec{\Omega}$ ($r = 1, 2, 3$ corresponds to x, y, z):

$$\vec{\Omega} = \frac{e}{mc} \left\{ \left(a + \frac{1}{\gamma} \right) \vec{B} - a \frac{\gamma}{\gamma+1} \left(\vec{\beta} \cdot \vec{B} \right) \vec{\beta} \right\}. \quad (236)$$

Now let us take into account that the target is located in the ring section where the magnetic field is absent. As a result, the system of equations (235) is conveniently split into two systems: one (see (237)) describes the behavior of the deuteron beam spin characteristics in that section and during the time interval, where the magnetic field \vec{B} is present, but the target is absent; the second (see (238)) describes the spin characteristics inside the target, where magnetic field is not applied:

$$\left\{ \begin{array}{l} \frac{d\vec{p}}{dt} = \left[\vec{p} \times \vec{\Omega} \right], \\ \frac{dp_{ik}}{dt} = - (\varepsilon_{jkr} p_{ij} \Omega_r + \varepsilon_{jir} p_{kj} \Omega_r) \end{array} \right. \quad (237)$$

$$\left\{ \begin{array}{l} \frac{d\vec{p}}{dt} = \frac{\chi}{2} (\vec{n}(\vec{n} \cdot \vec{p}) + \vec{p}) + \frac{\eta}{3} [\vec{n} \times \vec{n}'] - \frac{2\chi}{3} \vec{p} - \frac{\chi}{3} (\vec{n} \cdot \vec{n}') \vec{p}, \\ \frac{dp_{ik}}{dt} = \chi \left\{ -\frac{1}{3} + n_i n_k + \frac{1}{3} p_{ik} - \frac{1}{2} (n'_i n_k + n_i n'_k) + \frac{1}{3} (\vec{n} \cdot \vec{n}') \delta_{ik} \right\} + \\ + \frac{3\eta}{4} ([\vec{n} \times \vec{p}]_i n_k + n_i [\vec{n} \times \vec{p}]_k) - \frac{\chi}{3} (\vec{n} \cdot \vec{n}') p_{ik}, \end{array} \right. \quad (238)$$

where $\vec{n} = \vec{k}/k$, $\eta = -\frac{4\pi N}{k} \text{Re}d_1$, $n'_i = p_{ik} n_k$, $\chi = -\frac{4\pi v N}{k} \text{Im}d_1 = -vN(\sigma_1 - \sigma_0)$.

Suppose that at instant t_0 a target of thickness L is inserted into the beam's path. At this instant, particles possessing polarization vector \vec{p}_0 and polarization tensor $p_{ik}^{(0)}$ pass through

the boundary of the target. After beam entering the target, the spin characteristics \vec{p}_0 and $p_{ik}^{(0)}$ change due to interaction with the target in accordance with formulas (238). If the target is thin enough to make changes of vector and tensor polarization for a particle small, equations (238) can be solved using the perturbations theory. Therefore, from (237) and (238) for spin characteristics of a particle leaving the target $\vec{p}(t_0 + \tau)$ and $p_{ik}(t_0 + \tau)$, one can write:

$$\vec{p}(t_0 + \tau) = \vec{p}_0 + \frac{\chi}{2}(\vec{n}(\vec{n} \cdot \vec{p}_0) + \vec{p}_0)\tau + \frac{\eta}{3}[\vec{n} \times \vec{n}'_0]\tau - \frac{2\chi}{3}\vec{p}_0\tau - \frac{\chi}{3}(\vec{n} \cdot \vec{n}'_0)\vec{p}_0\tau, \quad (239)$$

$$\begin{aligned} p_{ik}(t_0 + \tau) &= p_{ik}^{(0)} + \chi \left[-\frac{1}{3} + n_i n_k + \frac{1}{3} p_{ik}^{(0)} - \frac{1}{2} (n'_{i0} n_k + n_i n'_{k0}) + \frac{1}{3} (\vec{n} \cdot \vec{n}'_0) \delta_{ik} \right] \tau + \\ &+ \frac{3\eta}{4} ([\vec{n} \times \vec{p}_0]_i n_k + n_i [\vec{n} \times \vec{p}_0]_k) \tau - \frac{\chi}{3} (\vec{n} \cdot \vec{n}'_0) p_{ik}^{(0)} \tau, \end{aligned} \quad (240)$$

where \vec{p}_0 is the beam polarization at instant t_0 , $n'_{i0} = p_{ik}^{(0)} n_k$, $p_{ik}^{(0)}$ are the components of polarization tensor at the same instant, τ is the time interval which the particle spends in the target.

The further evolution of \vec{p} and p_{ik} is again determined by the equations (235). After one revolution period T , a particle enters the target again possessing spin parameters $\vec{p}(t_0 + \tau + T)$ and $p_{ik}(t_0 + \tau + T)$, which have changed compared to their values at the time $(t_0 + \tau)$ due to the spin rotation in the magnetic field in the Nuclotron ring. These new values can be used as the initial conditions when solving the equations (238), i.e., one can use the solutions of (239) and (240) with the replacement of τ by $\tau + T$. This iterative process can be continued further. Solutions (239) and (240) are given for a single turn of a deuteron in a ring. The frequency of spin precession and that of cyclotron motion over the orbit are different (their ratio, i.e., spin tune is unconstrained). Therefore, after each particle turn in a ring those components of vector and tensor polarization, which lay in the orbit plane, differ from their initial values at the point where the target is supposed to be located. That is why the polarization vector component lying in the orbital plane of a particle rotating in a ring, when averaged over a time interval exceeding the rotation period, vanishes. However, the situation is entirely different for tensor polarization: the diagonal components of the polarization tensor, which arise due to the spin dichroism effect in an initially nonpolarized beam, do not vanish after averaging over time! (see section 6.9). Consequently, the arising spin dichroism effect does not depend on the ratio of the spin precession frequency to the cyclotron frequency (i.e. spin tune).

Hereinafter, it is more convenient to consider the evolution of polarization characteristics of a particle beam in an internal target of the Nuclotron in a different way.

6.8. Evolution of polarization characteristics of a particle beam in an internal target of Nuclotron

General dynamics of tensor polarization of particles in external fields is given in [128–130], while the general theory describing the motion of particles possessing tensor polarization in a target in the presence of electromagnetic fields is described in [37, 73].

To find the explicit expressions for the quantum-mechanical evolution operators, let us suppose that y -axis is directed along the direction of magnetic field \vec{B} , and z -axis is parallel to the momentum of a particle at the instant it enters a target. Then, the two parts of the Hamiltonian, which are responsible for the spin dynamics of the particle in the target (\hat{V}) can be written as follows [37, 73]:

$$\hat{V} = -\frac{2\pi\hbar^2 N}{M\gamma} \hat{f}(0), \quad (241)$$

and the Larmor precession in the storage ring (\hat{H}) reads as [132]:

$$\hat{H} = -\frac{e\hbar}{Mc} \left(\frac{g-2}{2} + \frac{1}{\gamma} \right) B_y \hat{S}_y. \quad (242)$$

Here N is the number of scatterers in cm^3 of the target, M is the mass of the incident particle, γ is its Lorentz factor, e is the particle charge and g is g -factor (for the deuteron $g \approx 0.86$), $\hat{f}(0) = d_0 + d_1 \hat{S}_z^2$ is the amplitude of coherent elastic forward scattering in the reference frame in which the target rests. Note that operator \hat{V} is non-Hermitian ($\hat{V} \neq \hat{V}^\dagger$) due to the presence of nonzero imaginary parts for parameters d_0 and d_1 .

Since τ is the time interval for a particle to pass through the target once, and $T - \tau \approx T$ is the time interval when the particle moves in the storage ring beyond the target ($\tau \ll T$), the evolution operators after passing each of two sections at a single turn are

$$\hat{U}_V = e^{-i\hat{V}\tau/\hbar} \quad (243)$$

and

$$\hat{U}_B = e^{-i\hat{H}T/\hbar}. \quad (244)$$

Then, the evolution operator for one turn in the storage ring is the product $\hat{U}_1 = \hat{U}_B \hat{U}_V$, and after n turns it is defined as $\hat{U}^{(n)} = \hat{U}_1^n$.

Using the following equalities valid for particles with spin $S = 1$, namely: $\hat{S}_z^4 = \hat{S}_z^2$ and $\hat{S}_y^3 = \hat{S}_y$, the evolution operators $\hat{U}_{B(V)}$ can be transformed as follows:

$$\hat{U}_B = \hat{\mathbf{I}} + (\cos(\phi) - 1) \hat{S}_z^2 + i \sin(\phi) \hat{S}_y \quad (245)$$

and

$$\hat{U}_V = e^{i\alpha} \left(\hat{\mathbf{I}} + (e^{i\zeta} - 1) \hat{S}_z^2 \right), \quad (246)$$

where

$$\phi = \frac{e}{mc\hbar} \left(\frac{g-2}{2} + \frac{1}{\gamma} \right) B_y T$$

is the spin rotation angle around the magnetic field direction per single turn in the storage ring,

$$\alpha = \frac{2\pi\hbar N}{M\gamma} d_0 \tau \quad (247)$$

is the complex quantity responsible for spin-independent beam attenuation, and

$$\zeta = \frac{2\pi\hbar N}{M\gamma} d_1 \tau \quad (248)$$

is responsible for spin dichroism and spin rotation after a single pass through the target.

Let the initial state of the particle beam in the storage ring be described by the density matrix $\hat{\rho}_0$. Then, the average values p_{ij} of the Cartesian components of the quadrupolarization operator [6, 73]

$$\hat{Q}_{ij} = \frac{3}{2S(S-1)} \left(\hat{S}_i \hat{S}_j + \hat{S}_j \hat{S}_i - \frac{2}{3} \hat{S}^2 \delta_{ij} \right) \quad (249)$$

and the average values p_i of the spin operator components \hat{S}_i read as follows:

$$p_{ij} = \frac{\text{Sp}(\hat{\rho}^{(n)} \hat{Q}_{ij})}{\text{Sp}(\hat{\rho}^{(n)})} \quad (250)$$

and

$$p_i = \frac{\text{Sp}(\hat{\rho}^{(n)} \hat{S}_i)}{\text{Sp}(\hat{\rho}^{(n)})}, \quad (251)$$

where $\hat{\rho}^{(n)} = \hat{U}^{(n)} \hat{\rho}_0 \hat{U}^{(n)+}$ is the density matrix after n turns of the particle in the accelerator. Since the relation $\hat{\rho}^{(n)} = \hat{U}_1 \hat{\rho}^{(n-1)} \hat{U}_1^+$ holds, the matrices $\hat{\rho}^{(n)}$ form an explicitly defined iterative sequence, which can be constructed using mathematical software packages.

In the case of an insignificant change of the state for an initially nonpolarized deuteron beam due to birefringence in a target ($|\zeta n| \ll 1$), the linear approximation over ζ can be used, evaluation $\hat{U}_V \approx e^{i\alpha \hat{I}} + i e^{i\alpha} \zeta \hat{S}_z^2$ is valid, and the evolution operator \hat{U} can be approximately expressed as follows:

$$\hat{U} \approx e^{i\alpha n} \hat{U}_B^n + i e^{i\alpha n} \left(\sum_{j=0}^{j=n-1} \hat{U}_B^j \zeta \hat{S}_z^2 \hat{U}_B^{-j} \right) \hat{U}_B^{n-1}. \quad (252)$$

Then, for $n \gg 1$, the component p_{zz} of quadrupolarization tensor reads as follows:

$$p_{zz} \approx -\frac{1}{3} n \text{Im} \zeta. \quad (253)$$

Discarding the rapidly oscillating term in (271), which comprises ratio of two sines, and using (185) and (248), one can get for p_{zz} the following:

$$p_{zz} \approx \frac{\Delta \sigma}{\sigma} \frac{N \sigma z}{6}. \quad (254)$$

where $\Delta \sigma = \sigma_0 - \sigma_{\pm 1}$ and $\sigma = \frac{2}{3} \sigma_{\pm 1} + \frac{1}{3} \sigma_0$ (for deuterons $\Delta \sigma > 0$), N is the number of atoms in cm^3 of the target. Note that for an internal target p_{zz} value is four times lower than for an external target at the same path length z for the particle in the target (see (254)).

Thus, deuterons passing through the Nuclotron internal target acquire tensor polarization. Quadrupolarization tensor component p_{zz} appears to be proportional to length z of the path, which a particle passes in the target. It should be noted that in the case of polarization measurements for deuterons interacting with the internal target, either deuterons scattered in the target or the products of their interactions with the nuclei in the internal target are detected, rather than the polarization of the transmitted beam. For the measurement with an external target, only the polarization of the transmitted beam is investigated. Particles, which came into collisions, are scattered and leave the beam. The average value of the path length z for a particle in the target is equal to the mean free path $1/N\sigma$. Therefore, the average value of p_{zz} for a single cycle of the Nuclotron:

$$\bar{p}_{zz} \approx \frac{\Delta \sigma}{6\sigma}. \quad (255)$$

Further discussion is based on applying formulas (250) and (255) to describe the spin dichroism phenomenon for deuterons in the internal target of the storage ring.

6.9. Dichroism effect for a deuteron beam moving in the Nuclotron with internal target

When conducting experiments with an internal target in the storage ring, it is necessary to consider the presence of a magnetic field, which leads to Larmor precession of the deuteron spin. This phenomenon leads to averaging the physical quantities and makes change in the

relationship between the diagonal components of tensor polarization of the beam. Due to precession, the average value of component p_{zz} becomes equal to the average value of component p_{xx} , and due to relation $p_{xx} + p_{yy} + p_{zz} = 0$, the average value of p_{yy} appears to be twice as large in magnitude as the average value of p_{zz} , with the signs of the average values of the components p_{zz} and p_{yy} being opposite. As it was already mentioned, in experiments with an internal target, the detector counts scattered particles, in contrast to experiments with an extracted beam, where particles transmitted through the target are detected. In contrast to the experiments with an external target, where a beam passes through the target only once, multiple beam turns N_{turns} in the Nuclotron contribute to path length z for a particle in the internal target, $z = N_{\text{turns}}l$, where l is the target thickness.

Figure 16 shows the dependence of the component p_{zz} on the path length in the target, which is expressed in the terms of L_{nuc} similar section 6.6.

Here, the value of $\Delta\sigma/\sigma$ is taken to be 0.01 ¹. For comparison, the same graph also shows the dependence of p_{zz} for the case of an external target, which is 6 times higher (compare (193) with (254) and (255)).

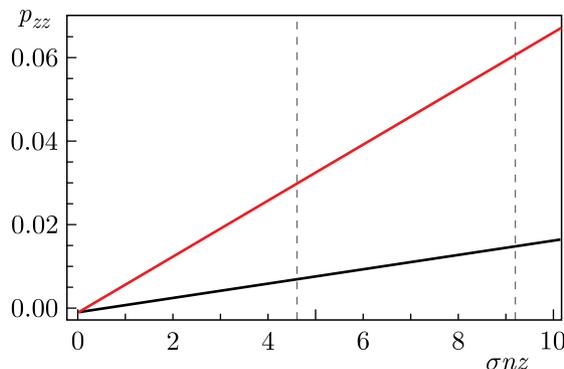


Figure 16. Dependence of the tensor polarization component p_{zz} on the path length for the particle in the target at $\Delta\sigma/\sigma = 0.01$. Black curve corresponds to the tensor polarization of the beam in experiments with an internal target, while red curve is for experiments with an external target. Vertical lines on the graphs correspond to beam attenuation by two and four orders of magnitude.

As an example, let us consider measurements with the use of polarimeter developed by [133], which comprises polyethylene CH_2 target of $10 \mu\text{m}$ thickness and the deuteron beam with 270 MeV energy (see also [143–145]). It is noteworthy that during a single cycle of the Nuclotron operation, which duration is about several seconds, the particle beam is completely absorbed in the target.

Several operation cycles are required to detect the effect. Preliminary analysis of the possibility to observe the spin dichroism effect at the polarimeter developed in JINR shows that accelerator operation during 20 to 30 hours enables effect acquisition [134].

The birefringence phenomena, which is described above and reveals itself as diverse effects, namely: spin and tensor polarization rotation around the momentum direction, spin oscillations, vector polarization conversion to tensor that and vice versa, as well as spin dichroism, should be taken into account when conducting precision experiments with either nonpolarized or polarized particle beams, since they lead to changes in the components of vector and tensor polarization of the beam and, thus, introduce systematic errors into the measurement results.

¹Analysis of deuteron dichroism experiments with momentum $5 \text{ GeV}/c$ [113, 115] shows that this value can be considerably higher ($\Delta\sigma/\sigma \approx 0.06$).

Table 4. Evaluation of deuteron dichroism for carbon and polyethylene (CH₂) targets.

Particle	Target	E , MeV	N_0	σ , b	$\Delta\sigma/\sigma$	$\langle p_{zz} \rangle$
d	CH ₂	270	10^{10}	0.8	0.06	$1 \cdot 10^{-2}$
d	CH ₂	270	10^{10}	0.8	0.01	$2 \cdot 10^{-3}$
d	C	3700	10^{10}	0.6	0.06	$1 \cdot 10^{-2}$
d	C	270	10^{10}	0.6	0.06	$1 \cdot 10^{-2}$

Evaluations of deuteron dichroism effect (acquired $\langle p_{zz} \rangle$ value) for carbon and polyethylene (CH₂) targets are given in Table 4. Evaluations are made for two $\Delta\sigma/\sigma$ values: the lowest expected $\Delta\sigma/\sigma = 0.01$ and that derived from measurements [115] $\Delta\sigma/\sigma = 0.06$.

6.10. About Possible Influence of Birefringence Effect on the Processes of Production (Photoproduction, Electroproduction) of Vector Mesons in Nuclei

Collision of high energy particles (proton, electron, γ -quanta, nucleus) with a nucleus yields a lot of hadronic processes inside nuclei, which are accompanied by the appearance of secondary particles with spin $S \geq 1$ (vector mesons, Ω^- hyperons, and so on). In particular, the processes of photoproduction (electroproduction) of vector mesons by nuclei have been studied because the photoproduction vertex for hadronic probes inside nuclei is well known and the analysis of the results is simple and more reliable.

Moreover, experiments demonstrate the production of both longitudinally (L) and transversally (T) polarized vector mesons and L/T-ratios depend on Q^2 [135, 136].

According to [137], treating photoproduction of vector mesons inside nuclei and their rescattering via strong interactions within the framework of the Glauber multiple scattering theory allows one to consider many peculiarities of the process, which are important for understanding unconventional effects, such as color transparency. According to the analysis given in [138], theoretical consideration of the processes of rescattering of produced particles inside the nucleus must take into account the possible influence of the spin of produced particle on rescattering inside the nucleus and, therefore, the influence of birefringence effect (spin dichroism and rotation) on characteristics of the produced vector mesons.

In particular, as is known from the previous sections, birefringence phenomenon appears when a particle with spin $S \geq 1$ passes through matter. Specifically, the effect of spin dichroism arises.

For a particle produced inside a nucleus and moving through the nuclear matter, the conception of the refraction index can also be applied [15, 139].

As a result, the particle produced inside the nucleus undergoes refraction described by the spin-dependent index of refraction [138] (see also (1)):

$$\hat{N} = 1 + \frac{2\pi\rho(\vec{r})}{k^2} \hat{f}(0),$$

where $\rho(\vec{r})$ is the density of scatterers in matter (the number of scatterers in 1 cm³), k is the particle wave number, $\hat{f}(0)$ is used to denote the amplitude of zero-angle elastic coherent scattering of a particle by a scattering center; this amplitude is an operator acting in the particle spin space.

Refraction of particles in matter (either conventional or nuclear) implies the existence of an optical pseudopotential depending on particle spin:

$$\hat{V}_{\text{eff}} = -\frac{2\pi\hbar^2\rho(\vec{r})}{m\gamma}\hat{f}(0), \quad (256)$$

where m is the particle mass and γ is its Lorentz factor.

In the case we are concerned with, for example, of production of particles with spin 1 (vector mesons), the amplitude of zero-angle elastic coherent scattering by nonpolarized nuclear nucleons can be expressed in the following general form:

$$\hat{f}(0) = d + d_1(\vec{S}\vec{n})^2, \quad (257)$$

where \vec{S} is the operator of the particle spin, $\vec{n} = \frac{\vec{k}}{k}$ is the unit vector along the particle momentum. The angle of spin rotation is determined by $\text{Re } d_1$, while $\text{Im } d_1$ describes dichroism.

The occurrence of spin dichroism means that spin features of a particle produced in nuclear matter (another particle with spin ≥ 1) will differ from spin properties of the particle produced by a stand-alone nucleon (which does not compose the nucleus), which brings about, in particular, the change in the L/T -ratio [138].

To describe rescattering processes in the energy range, where $\text{Re } f(0) \ll \text{Im } f(0)$, the expressions obtained in [137] can be used. But the total cross-sections of vector meson production given there should be replaced by $\sigma_{M=\pm 1}$ or $\sigma_{M=0}$.

In the case when $\text{Re } f(0)$ is comparable with $\text{Im } f(0)$ or larger, the additional analysis is required.

The detailed analysis of dichroism effect in the case of vector meson production by nuclei was recently presented in [140].

It should be mentioned that, in the general case, two correlations are present in photoproduction. Correlation $[\vec{e}^*\vec{e}]\vec{S}$ (where \vec{S} is the spin operator of a produced particle) is sensitive to circular polarization of photons and the produced particle has vector polarization. Correlation $(\vec{e}\vec{S})^2$, which is sensitive to the linear polarization of photons, corresponds to production of particle with tensor polarization. Due to the birefringence effect, the produced particles are absorbed differently by the nucleus. Therefore, the yield of vector-mesons depends on the photon polarization, i.e., the production cross-sections are different for different polarizations of the incident photons $\sigma_{\text{circ}} \neq \sigma_{\text{lin}}$ [138].

7. “Optical” spin rotation and birefringence of particles at NICA

Recall now that matter (gas, jet target) can be contained in a storage ring. As shown in the previous sections, a particle moving in matter undergoes the action of pseudomagnetic and pseudoelectric fields, which leads to “optical” spin rotation and birefringence effects. As a result, the presence of a target influences the particle spin motion in a storage ring. It should be recalled that the particle’s refractive index in matter has the form (see also (1)):

$$n = 1 + \frac{2\pi N}{k^2}f(0),$$

where N is the number of particles per cm^3 and k is the wave number of the particle incident on the target, $f(0)$ is the coherent elastic zero-angle scattering amplitude.

Let us consider particle refraction on the vacuum–medium boundary (see Fig. 17).

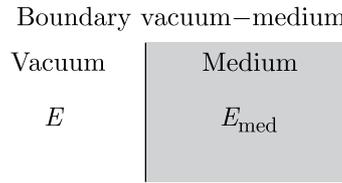


Figure 17. Particle energy E in vacuum is not equal to particle energy E_{med} in a medium.

The wave number of the particle in the vacuum is denoted k . The wave number of the particle in the medium is $k' = kn$. As is evident, the particle momentum in the vacuum $p = \hbar k$ is not equal to the particle momentum in the medium. Therefore, the particle energy in the vacuum $E = \sqrt{\hbar^2 k^2 c^2 + m^2 c^4}$ is not equal to the particle energy in the medium $E_{\text{med}} = \sqrt{\hbar^2 k^2 n^2 c^2 + m^2 c^4}$.

As stated above, the energy conservation law immediately necessitates the particle in the medium to have an effective potential energy V_{eff} . This energy can be easily found from the relation

$$E = E_{\text{med}} + V_{\text{eff}},$$

i.e.,

$$V_{\text{eff}} = E - E_{\text{med}} = -\frac{2\pi\hbar^2}{m\gamma} N f(E, 0) = (2\pi)^3 N \mathcal{T}(E), \tag{258}$$

where $\mathcal{T}(E)$ is the \mathcal{T} -matrix [15] and

$$f(E, 0) = -(2\pi)^2 \frac{E}{c^2 \hbar^2} \mathcal{T}(E) = -(2\pi)^2 \frac{m\gamma}{\hbar^2} \mathcal{T}(E).$$

7.1. Mutual refraction of colliding beams

Until now we have discussed the rest target. But a bunch, which moves in a storage ring, can also be considered as a target. Therefore, (258) and (258) should be generalized for this case [37, 73].

Let E_1 and γ_1 denote the energy and Lorentz-factor of the particles of the first beam in the rest frame of the storage ring, and E_2 and γ_2 denote those of the particles of the second beam.

Bearing in mind that the wave phase in the medium is Lorentz-invariant, one can find it in the following way.

Let the second beam be at rest in the reference frame; then the refraction index can be expressed in the conventional form (258):

$$n'_1 = 1 + \frac{2\pi N'_2}{k_1'^2} f(E'_1, 0), \tag{259}$$

where $N'_2 = \gamma_2^{-1} N_2$ is the density of bunch 2 in its rest frame, and N_2 is the density of the second bunch in the storage ring frame, k'_1 and E'_1 are the wave number and energy of particles of the first bunch in the rest frame of bunch 2, respectively. Let L denote the length of bunch 2 in its rest frame; then $L = \gamma_2 l$, where l is the length of this bunch in the storage ring frame.

The interaction of a particle from bunch 1 (particle 1) with particles of bunch 2 causes a change in the phase of the wave:

$$\phi = k'_1(n'_1 - 1)L = \frac{2\pi N'_2}{k_1'} f(E'_1, 0) L = \frac{2\pi N_2}{k_1'} f(E'_1, 0) k'_1 l. \tag{260}$$

It is known [15] that the ratio

$$\frac{f(E'_1, 0)}{k'_1}$$

is invariant, i.e.,

$$\frac{f(E'_1, 0)}{k'_1} = \frac{f(E_1, 0)}{k_1},$$

where $f(E_1, 0)$ is the amplitude of elastic coherent forward scattering of particle 1 by moving particles of e bunch 2 in the rest frame of the storage ring.

As a result

$$\phi = \frac{2\pi N_2}{k_1} f(E_1, 0) \cdot l = \frac{2\pi N_2}{k_1} f(E_1, 0) \cdot v_{\text{rel}} \cdot t, \quad (261)$$

where v_{rel} is the velocity of the relative motion of particle 1 and bunch 2 (for opposing motion $v_{\text{rel}} = (v_1 + v_2)(1 + \frac{v_1 v_2}{c^2})^{-1}$), and t is the time of interaction of particle 1 with bunch 2 in the rest frame of the storage ring. The particle with velocity $v_1 = \frac{\hbar k_1 c^2}{E_1}$ passes the distance $z = v_1 \cdot t$ over time t . Note that the distance z differs from the length of bunch 2, since it moves. Expression (261) can be rewritten as:

$$\phi = \frac{2\pi N_2}{k_1} f(E_1, 0) \frac{v_{\text{rel}}}{v_1} z = k_1 (n_1 - 1) z, \quad (262)$$

where the index of refraction of particle 1 by the beam of moving particles 2 is:

$$n_1 = 1 + \frac{2\pi N_2}{k_1^2} \frac{v_{\text{rel}}}{v_1} f(E_1, 0). \quad (263)$$

When $v_2 = 0$, expression (263) converts to the conventional view (258).

Thus, the effective potential energy V_{eff} being acquired by particle 1 when it collides with the particles of bunch 2 can be written as follows:

$$\begin{aligned} V_{\text{eff}} &= E_1 - E_{1 \text{ med}} = E_1 - \sqrt{p_1^2 c^2 n_1^2 + m_1^2 c^4} \\ &= -2\pi \hbar^2 N_2 v_{\text{rel}} \frac{f(E_1, 0)}{p_1} = -2\pi \hbar^2 N_2 v_{\text{rel}} \frac{f(E'_1, 0)}{p'_1}. \end{aligned} \quad (264)$$

Therefore,

$$V_{\text{eff}} = -\frac{2\pi \hbar^2 N_2}{m_1 \gamma_1 \gamma_2} f(E'_1, 0) = (2\pi)^3 N_2 \mathcal{T}(E'_1), \quad (265)$$

where $E'_1 = m_1 c^2 \gamma_1 \gamma_2$ is the energy of particle 1 in the rest frame of bunch 2, p_1 denotes the momentum of particle 1 in the storage ring frame, while p'_1 is the momentum of particle 1 in the rest frame of bunch 2 ($p'_1 = \frac{E'_1 v'_1}{c^2}$) and $v'_1 = v_{\text{rel}}$.

When obtaining (264), $|n_1 - 1| \ll 1$ was used.

Let us consider now a particle with the nonzero spin. In this case, the amplitude of the zero-angle scattering depends on the particle spin and, as a consequence, the index of refraction depends on the particle spin and can be written as:

$$\hat{n}_1 = 1 + \frac{2\pi N_2}{k_1^2} \frac{v_{\text{rel}}}{v_1} \hat{f}(E_1, 0), \quad (266)$$

where $\hat{f}(E_1, 0) = \text{Tr} \hat{\rho}_J \hat{F}(0)$, the operator of the forward scattering amplitude $\hat{F}(0)$ acts in the combined spin space of the particle and scatterer spin, $\hat{\rho}_J$ is the spin density matrix of the scatterers.

According to the above, a particle in matter possesses some effective potential energy V_{eff} . If the amplitude $\hat{f}(0)$ of particle scattering depends on the particle spin, then the effective energy depends on the spin orientation:

$$\hat{V}_{\text{eff}} = -\frac{2\pi\hbar^2 N_2}{m_1 \gamma_1 \gamma_2} \hat{f}(E'_1, 0). \quad (267)$$

Therefore, all the above considered quasi-optical phenomena (spin rotation and spin dichroism) arise at collisions of particles and nuclei beams. For example (see section 6.4), scattering amplitude $\hat{f}(0)$ of a particle with spin $S = 1$ (for example, deuteron) in a nonpolarized target depends on the particle spin and can be written as:

$$\hat{f}(E'_1, 0) = d + d_1 (\hat{\vec{S}} \cdot \vec{n})^2, \quad (268)$$

where $\hat{\vec{S}}$ is the deuteron spin operator and \vec{n} is the unit vector along the deuteron momentum \vec{k} .

Substituting (268) to (267), one can obtain for a particle with spin $S = 1$:

$$\hat{V}_{\text{eff}} = -\frac{2\pi\hbar^2}{m_1 \gamma_1 \gamma_2} N_2 \left(d + d_1 (\hat{\vec{S}} \cdot \vec{n})^2 \right). \quad (269)$$

Let the quantization axis z be directed along \vec{n} and M denote the magnetic quantum number. Then, for a particle in the eigenstate of the operator of spin projection onto the z -axis \hat{S}_z , the effective potential energy can be written as:

$$\hat{V}_{\text{eff}} = -\frac{2\pi\hbar^2}{m_1 \gamma_1 \gamma_2} N_2 (d + d_1 M^2). \quad (270)$$

According to (270), splitting of the deuteron energy levels in matter is similar to splitting of the atom energy levels in an electric field due to the quadratic Stark effect. Therefore, the above effect could be considered as caused by splitting of the particle spin levels in the pseudoelectric nuclear field of matter.

As a result, the spin of a particle with $S \geq 1$, which participates in collisions of counter-propagating beams, rotates and oscillates (birefringence effect) (see section 6.4) due to interaction with the pseudoelectric nuclear field of bunches.

7.2. Evaluations of spin dichroism effect at beam collisions at NICA

At the collision of two initially nonpolarized deuteron beams at NICA, both beams acquire tensor polarization $p_{zz} = -(p_{xx} + p_{yy}) \neq 0$ (see Fig. 18). If siberian snakes are not available, rotation of polarization occurs at each turn.

Let us evaluate the acquired tensor polarization p_{zz} at typical NICA parameters. According to [141], the cycle of NICA operation is described as follows: “After the beginning of the beam collisions at a maximal luminosity L_0 , the data taking continues for a period T_1 that is lower than the luminosity decay time τ_L (which, in turn, is much longer than the polarization decay time τ_P). During this time, the RF system of the collider provides an acceptable longitudinal size of the colliding bunches. After that, the existing beams are dumped and the accelerator complex spends time T_2 to accelerate and store a new portion of particles. Assuming $T_1 = 2$ hours,

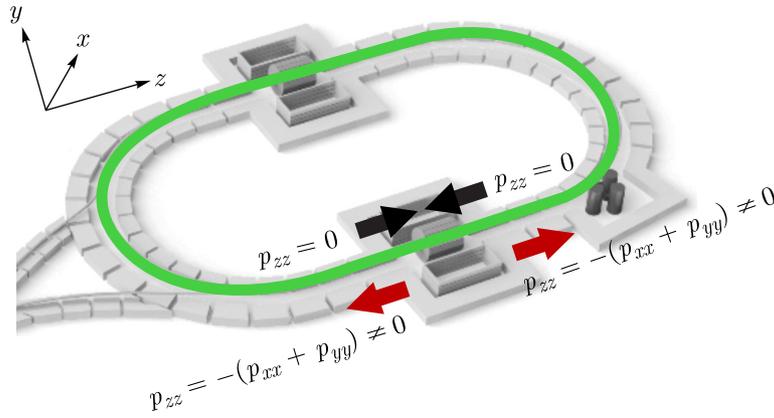


Figure 18. Spin dichroism effect at beams collisions at NICA without siberian snakes.

$T_2 = 1$ hour and $\tau_L = 6$ hours, the effective luminosity L_{eff} averaged over the cycle is about $0.6 \times L_0$ ”.

The tensor polarization acquired by a deuteron beam during period T_1 is given by:

$$p_{zz} = \frac{1}{6} \frac{\Delta\sigma}{\sigma} \frac{LT_1}{N_b}, \quad (271)$$

where L is the luminosity, N_b is the number of deuterons per beam, σ is the total cross-section for dd collisions $\sigma = \frac{2}{3}\sigma_{\pm 1} + \frac{1}{3}\sigma_0$ and $\Delta\sigma$ is the difference between scattering cross-sections for spin projections with $m = 0$ and $m = \pm 1$ i.e. $\Delta\sigma = \sigma_0 - \sigma_{\pm 1}$. Typical parameters for deuteron beams collision at NICA [141] are used: number of particles per beam $N_b = 1.3 \cdot 10^{13}$, luminosity $L = 4 \cdot 10^{31} \text{ cm}^{-2}\text{s}^{-1}$ and data taking period $T_1 = 2$ hours. Total cross-section of dd collisions is supposed to be as high as $\sigma = 0.2$ barn. Two values of $\Delta\sigma/\sigma$ ratio are used for calculation: 0.01 as the lowest expected and 0.06 as reported in [115]. Therefore, the acquired tensor polarization p_{zz} is expected to be within interval $0.75 \cdot 10^{-5}$ and $0.5 \cdot 10^{-4}$. To understand whether the expected p_{zz} value is observable, one should analyze statistics on the number of events during a data-taking period (the systematic errors, which also affect the possibility to observe the effect, should be analyzed at real experiment planning). Several operation cycles are required to detect the effects. Analysis presented in [123, 125, 142] demonstrate that dozens and hundreds of hours are required to observe the effect — the specific experiment duration depends on particular cross-sections and analyzing powers for the certain energy of colliding beams.

The change in the number of beam particles occurs due to both elastic and inelastic collisions (reactions) (other loss mechanisms also exist, for example, intra-beam scattering [141]). Let us henceforward study inelastic processes. The influence of elastic processes is hardly accountable, since part of elastically scattered particles are scattered at a small angle and do not leave the orbit, thus, making no contribution to the change of beam polarization. For all evaluations, beam parameters are taken from [141].

Suppose σ_r to be the sum of cross-sections for all reactions sensitive to tensor polarization, which value is of the order of the total cross-section ($\sigma_r \sim \sigma$). The analyzing powers are denoted by A_r . The number of reactions during period T_1 reads as follows:

$$N_r = \sigma_r LT_1. \quad (272)$$

Since some deuterons in the beam moving in the Nuclotron acquire tensor polarization, the actual number of reactions changes with time.

The difference in numbers of events N_{diff} , which can be observed at $p_{zz} = 0$ and $p_{zz} \neq 0$, i.e., the part of events caused by nonzero p_{zz} , reads as follows:

$$N_{\text{diff}} \sim A_r N_r p_{zz} = A_r \sigma_r L T_1 p_{zz}. \quad (273)$$

The statistical error for the number of reactions is proportional to the root of number of reactions as follows:

$$\Delta N_r \sim \sqrt{\sigma_r L T_1}. \quad (274)$$

One could improve signal-to-noise ratio by increasing the observation time T , which is composed of periods T_1 and T_2 repeated as many times n_c as many operation cycles are made:

$$T = n_c(T_1 + T_2). \quad (275)$$

The number of reactions N_r^T during period $T = n_c(T_1 + T_2)$ reads as follows:

$$N_r^T = \sigma_r L T_1 n_c = \sigma_r L T_1 \frac{T}{(T_1 + T_2)} = \sigma_r L \frac{T_1}{(T_1 + T_2)} T. \quad (276)$$

Therefore,

$$N_{\text{diff}}^T \sim A_r \sigma_r p_{zz} L \frac{T_1}{(T_1 + T_2)} T. \quad (277)$$

Statistical error can be evaluated as

$$\Delta N_r^T \sim \sqrt{\sigma_r L \frac{T_1}{(T_1 + T_2)} T}. \quad (278)$$

Signal-to-noise ratio reads as follows:

$$\frac{N_{\text{diff}}^T}{\Delta N_r^T} = A_r p_{zz} \sqrt{\sigma_r L \frac{T_1}{(T_1 + T_2)} T}. \quad (279)$$

For example, 1 month observation time at $\frac{\Delta\sigma}{\sigma} = 0.01$ gives

$$\frac{N_{\text{diff}}}{\Delta N_r} \sim \frac{1}{2} A_r \cdot 10^3 \quad (280)$$

and even more promising results are obtained for $\frac{\Delta\sigma}{\sigma} = 0.06$:

$$\frac{N_{\text{diff}}}{\Delta N_r} \sim A_r \cdot 10^2. \quad (281)$$

The assumption that the analyzing power is as high as $A_r = 0.1$ ensures the effect can be measured, though premise $A_r = 0.01$ does not confine this possibility. Evaluations of the acquired tensor polarization p_{zz} at dd collisions are given by Table 5.

Note that the presence of Siberian snakes or operation in the spin transparency (ST) mode both increases the magnitude of dichroism effect and enables to observe the effects of coherent spin rotation and conversion of vector polarization to tensor one and vice versa, as well as spin rotation in pseudomagnetic field in the case of polarized beams.

Table 5. Spin dichroism effect: evaluations for dd collisions at NICA without the Siberian snake at luminosity as high as $L = 4 \cdot 10^{31} \text{ cm}^{-2} \cdot \text{s}^{-1}$ and $T = T_1 = 2$ hours.

Part.	$\sigma, \text{ b}$	$\Delta\sigma/\sigma$	N_b	N_{diff}	ΔN_r	p_{zz}
d-d	0.2	0.01	$1.3 \cdot 10^{13}$	$2 \cdot 10^6 A_r$	$2 \cdot 10^5$	$0.75 \cdot 10^{-5}$
d-d	0.2	0.06	$1.3 \cdot 10^{13}$	$10^7 A_r$	$2 \cdot 10^5$	$0.5 \cdot 10^{-4}$

8. Conclusion

Quasi-optical phenomenon of nuclear spin precession of particles (nuclei) in the pseudo-magnetic field of matter with polarized spins and the phenomenon of birefringence of particles (nuclei) with spin $S \geq 1$ can be observed at the Nuclotron-M/NICA complex. Studies of these phenomena allow measurement of the spin-dependent part of the amplitude of elastic coherent forward scattering. The above mentioned phenomena are not caused by strong interactions only. According to sections 5 and 6, the T-odd P-odd, T-odd P-even, and T-even P-odd interactions also contribute. Limits on the value of these contributions at energies available at the Nuclotron-M/NICA complex can be obtained by investigating all these phenomena. When studying polarized particle collisions, it is necessary to consider possible influences of quasi-optical phenomena of spin rotation and spin dichroism caused by nuclear precession and birefringence.

In the near future, it appears feasible to prepare and carry out the following experiments at the Nuclotron-M/NICA complex:

1. Investigation of the spin dichroism effect for deuterons at the Nuclotron with the internal target.
2. Observation and study of rotation of spin and tensor polarization, as well as conversion of vector polarization to tensor for deuterons passing through a nonpolarized target using the extracted beam at the Nuclotron.
3. Observation of tensor polarization acquiring by heavy nuclei (for example, Ne) when passing through an external target at the Nuclotron.
4. Preparing such experiments at NICA.

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Conflicts of Interest

The author declares no conflicts of interest.

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