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ZFITTER: Theoretical calculations and precision tests of the Standard Model

A. B. Arbuzov*[1,2], L. V. Kalinovskaya[1,3], A. G. Olshevskiy[1,3], and V. L. Yermolchyk[1,2]

[1] *Joint Institute for Nuclear Research, Dubna, Russia*
[2] *Dubna State University, Dubna, Russia*
[3] *Moscow State University, Moscow, Russia*

Abstract

The development of the ZFITTER computer code is described in the context of high-precision tests of the Standard Model during the LEP era. The features of the code that allowed it to become a standard tool for the theoretical interpretation of electroweak observables are analyzed. Prospects for further development of ZFITTER and its contribution to research projects at future electron–positron colliders are discussed. Numerical illustrations are provided of the effects of parameter shifts and the addition of new results for higher-order radiative corrections.

Keywords: the Standard Model, radiative corrections, electroweak interactions, ZFITTER

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This work is dedicated to our teacher, colleague,
and friend Dmitry Yurievich Bardin.

1. Introduction

Experiments at the Large Electron–Positron (LEP) Collider at CERN (1989–2000) made a major step in the development of high-energy physics (HEP). LEP initially was running at energies around 91 GeV to produce and study Z bosons and later at an increased energy to produce pairs of W bosons, reaching the maximum collision energy of about 209 GeV. The LEP experiments studied electroweak (EW) interactions in detail, confirmed the existence of three generations of matter particles, and tested properties of the Standard Model (SM) with an unprecedented precision. It was a breakthrough time for pioneers in the development of computing technology and data analysis methods that influenced many future experiments in various fields of physics.

A vast set of measurements of realistic observables was performed at the LEP experiments (ALEPH, DELPHI, L3, and OPAL), the SLD experiment using a polarized beam at the Stanford Linear Collider (SLC) and at the Tevatron. Cross sections, their ratios, and asymmetries were studied with a very high accuracy of the order of one *per mille* or even better, which

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*Corresponding author e-mail address: `arbuzov@theor.jinr.ru`

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provided sensitivity to quantum electroweak and QCD effects in higher orders of perturbation theory. The LEP Electroweak Working Group reduced the measurement results to a convenient set of 17 physical quantities, so-called pseudo-observables [1]. A comprehensive review on radiative corrections to observables at the Z peak was published in the CERN Yellow Report [2]. At the same time, it was necessary to create computer codes with this theoretical base, which could then be used to analyze data and fit the SM parameters. The conceptual elements of the theoretical basis were developed over a period of about 20 years (see Section 2), followed by the phase of transforming the established basis into the precise theoretical predictions for concrete observables. The painstaking work of experienced groups to create computer codes for theoretical support was accompanied by Working Group meetings to carefully compare the outputs of the developed codes. A significant effort was made to create codes containing the most comprehensive library of the relevant QED, electroweak and QCD radiative corrections and there were several different approaches to evaluate theoretical uncertainties. The corresponding working meetings focused on careful comparisons, cross checks, and their improvement [3]. In the beginning, there were about 30 competing codes, but for the actual LEP1 data analysis only five of them survived, namely BHM [4], WOH [5], LEPTOP [6], TOPAZ0 [7–9], and ZFITTER [10, 11]. And by the end of the LEP1 operation, only the last two codes remained in the game. Since then, during the LEP era, theoretical estimates of electroweak observables have been produced by the TOPAZ0 and ZFITTER programs.

Perturbative calculations within the Standard Model are usually organized in such a way that all SM parameters that are relevant for a given observable are determined using a limited number of input quantities. As a rule, each pseudo-observable is calculated based on five essential input parameters: the running electromagnetic coupling constant (α_{em}), the strong coupling constant (α_s) evaluated at the Z boson pole, the Z boson mass (M_Z), the top quark mass (m_t), and the Higgs boson mass (M_H). The Fermi constant (G_F) is employed to derive the W boson mass. The dependence on the masses of the top quark and the Higgs boson manifests itself below the production thresholds of these particles through electroweak loop corrections.

Accurate measurements impose constraints on the input parameter set (IPS), which allows one to extract information about unknown parameters or to refine those that were previously measured insufficiently accurately. This procedure differs from making direct predictions as described earlier. The discoveries of the W and Z bosons, as well as of the top quark, serve as examples of how theoretical constraints have allowed us to estimate the masses of previously unknown particles. A similar approach was applied to the Higgs boson H. It is important to understand that the Standard Model does not directly predict its parameters, but only provides certain relations between them.

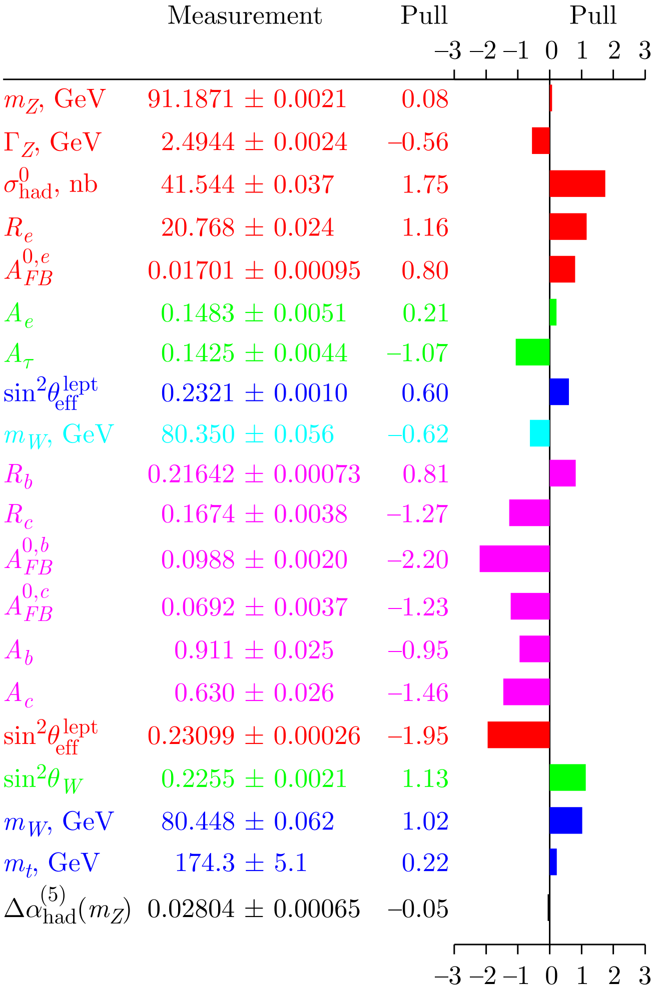
ZFITTER calculates cross sections for the reaction $e^+e^- \to f\bar{f}$ at energies around the Z peak. It is neither a dedicated fitting program, nor a Monte Carlo event generator. Nevertheless, the efficient modeling of phenomena at the Z peak, combining exact electroweak loop corrections and one-dimensional representations of the photonic corrections, provided the basis for a preferred use of ZFITTER in data fitting. ZFITTER contains the SM library (also called the weak library) which computes form factors and some other quantities, whose central part is the DIZET computer program. The other part consists of QED corrections from additional real or virtual photon emission.

The procedure for comparing experimental measurements with the SM predictions can be symbolically written as follows:

$$O_i^{\text{exp}}\,(\text{measured}) \leftrightarrow O_i^{\text{theor}}\,(\text{calculated as a function of IPS})\,. \tag{1}$$

The various parameters are experimentally known with different precision. For instance, the precision in the measurements of masses ranges from 10^{-7} for the electron mass m_e to the existence of only the lower and upper limits for the Higgs boson mass M_H (before LHC).

The status of the Standard Model that was established at LEP can be effectively represented by the so-called pulls shown in Figure 1 where one can see the differences between the values of the pseudo-observables (POs), obtained from the global fit of experimental data, and the corresponding values from dedicated experimental measurements. There are only a few instances when the pulls are of about two standard deviations, while the overall agreement is exceptionally strong. It is important to highlight that the achieved precision is of the order of $\sim 10^{-3}$, and managing the experimental systematic uncertainties at this level was highly challenging.



	Measurement	Pull
m_Z, GeV	91.1871 ± 0.0021	0.08
Γ_Z, GeV	2.4944 ± 0.0024	–0.56
$\sigma^0_{\rm had}$, nb	41.544 ± 0.037	1.75
R_e	20.768 ± 0.024	1.16
$A^{0,e}_{FB}$	0.01701 ± 0.00095	0.80
A_e	0.1483 ± 0.0051	0.21
A_τ	0.1425 ± 0.0044	–1.07
$\sin^2\theta^{\rm lept}_{\rm eff}$	0.2321 ± 0.0010	0.60
m_W, GeV	80.350 ± 0.056	–0.62
R_b	0.21642 ± 0.00073	0.81
R_c	0.1674 ± 0.0038	–1.27
$A^{0,b}_{FB}$	0.0988 ± 0.0020	–2.20
$A^{0,c}_{FB}$	0.0692 ± 0.0037	–1.23
A_b	0.911 ± 0.025	–0.95
A_c	0.630 ± 0.026	–1.46
$\sin^2\theta^{\rm lept}_{\rm eff}$	0.23099 ± 0.00026	–1.95
$\sin^2\theta_W$	0.2255 ± 0.0021	1.13
m_W, GeV	80.448 ± 0.062	1.02
m_t, GeV	174.3 ± 5.1	0.22
$\Delta\alpha^{(5)}_{\rm had}(m_Z)$	0.02804 ± 0.00065	–0.05

Figure 1. Pulls for POs. The pull is defined as the difference between the measurement and the SM prediction calculated for the central values of the fitted SM IPS [$\alpha(M_Z^2) = 1/128.878$, $\alpha_s(M_Z^2) = 0.1194$, $M_Z = 91.1865$ GeV, $m_t = 171.1$ GeV] divided by the experimental error [12].

2. Curriculum vitae of ZFITTER

This section provides a summary of the academic and professional history (background), encompassing research activities, publications, presentations, and awards of the ZFITTER project.

Here it is worth noting a comprehensive review [13] of some of the main developers of the ZFITTER project. This work provides an in-depth analysis of the historical development and evolution of the ZFITTER code and contains an almost complete bibliography of theoretical calculations of radiative corrections for e^+e^- annihilation processes.

The groundbreaking works by T. Kinoshita and A. Sirlin, within the framework of the $V-A$ theory of weak charged-current interactions [14, 15], refined the predictions of this theory. In a series of articles, they calculated radiative corrections to Michel's formula [16] for the muon decay spectrum to the first order in α [17, 18]. The calculations within the four-fermion $V-A$ theory suffer from ultraviolet divergences due to the non-renormalizable nature of this model. This problem could be solved only within the framework of a more fundamental theory. Later M. Veltman and A. Sirlin worked on the $V-A$ theory of weak interactions and supported it with high-precision calculations.

The key step in the calculation of the full set of one-loop radiative corrections to the $e^+e^- \to \mu^+\mu^-$ cross section within the electroweak framework was made by M. Veltman and G. Passarino [19]. They developed methods that enabled more general one-loop electroweak calculations to become practically feasible and established a technique to reduce all Feynman loop integrals in these calculations to a small set of so-called master integrals. After calculating these basic integrals, the procedure for calculating radiative corrections essentially reduces to purely algebraic transformations, although it remains quite complex. The subsequent key advancement was the comprehensive evaluation by M. Veltman and G.'t Hooft [20] of one-loop Feynman integrals involving propagators of massive particles.

In the lead-up to LEP, a solid platform for understanding the renormalization procedure and calculating the QED/EW loop corrections was established. There was also a burst of works devoted to the evaluation of high-order effects for elastic fermion scattering.

The ZFITTER project was originally called ZBIZON [21, 22], but the name was later changed to ZFITTER, and it is still known by that name today. Paper [23] presented the first analysis of two LEP observables: the weak mixing angle $\sin^2_W$ and the Z boson mass, together with their dependence on the unknown masses of the top quark and the Higgs boson within the Standard Model. The title of this work “Hunting the Hidden Standard Higgs” became the inspiration for the further development of ZFITTER during the LEP era.

The main body of the ZFITTER program relies on the analytical results, presented in [22, 24, 25] for the QED part and in [26–30] for the electroweak physics part, and on the library of electroweak corrections DIZET [26]. Selected formulae referenced here are exclusively presented in [11, 31]. These works formed the basis of the ZFITTER project. This basis was provided by the work of researchers from the field of high-energy physics, with a long history of development of theoretical support, up to new experimental results and active collaboration with the international scientific community.

The ZFITTER program offers a flexible treatment of the complete $\mathcal{O}(\alpha)$ QED corrections along with some higher-order contributions by using three computational chains, each imposing different realistic restrictions on the radiated photon phase space. Numerical integrations are limited to one dimension at most. The code includes full $\mathcal{O}(\alpha)$ weak loop corrections supplemented by selected higher-order terms. The program calculates POs: Δr, the Z boson width, differential and total cross sections, integrated forward–backward asymmetries, left–right asymmetries, and final-state polarization effects in τ pair production. Various interfaces enable fits with different sets of free parameters.

After calculating the pseudo-observables, one has to pass to realistic observables, such as various cross sections and asymmetries that include experimental cuts. To achieve this, we need to construct the so-called improved Born approximation (IBA). The latter is sometimes referred to as doubly deconvoluted observables because they are free from initial-state QED and final-state QED⊗QCD corrections. These two groups of corrections correspond to two separately gauge-invariant subsets of diagrams. All other remaining diagrams, including purely electroweak and internal QCD corrections, contribute to the IBA.

2.1. Improved Born approximation

Within the ZFITTER framework, the weak form factors $\rho_{ef}, \kappa_e, \kappa_f, \kappa_{ef}$ are introduced as the key elements of the IBA approach. This method was first introduced in [32] for the Z boson decay. In papers [10, 26], based on [33, 34], the concept was generalized to $2 \to 2$ scattering. Detailed formulae used in this context can be found in [11, 35]. Let us start by briefly reviewing the key aspects of the one-loop electroweak form factors and the IBA. We begin with the Born amplitudes for the process $e^+e^- \to f\bar{f}$, which is described by two Feynman diagrams with γ and Z exchange.

The photon exchange amplitude has unique *vector* ⊗ *vector* structures

$$A_\gamma^{\text{Born}} = \frac{e^2 Q_e Q_f}{s} \gamma_\mu \otimes \gamma_\mu. \tag{2}$$

The Z exchange amplitude can be written in the VA basis as

$$A_Z^{\text{Born}} = \frac{e^2}{4 s_W^2 c_W^2}\, \chi_z(s) \gamma_\mu \left(v_e + a_e \gamma_5\right) \otimes \gamma_\mu \left(v_f + a_f \gamma_5\right), \tag{3}$$

or more conveniently in the LQ basis in the form

$$A_Z^{\text{Born}} = \frac{e^2}{4 s_W^2 c_W^2}\, \chi_z(s) \gamma_\mu \left[I_e^{(3)}(1+\gamma_5) - 2Q_e s_W^2\right] \otimes \gamma_\mu \left[I_f^{(3)}(1+\gamma_5) - 2Q_f s_W^2\right], \tag{4}$$

where symbol ⊗ is used in the following short-hand notation:

$$\bar{u}(p_+)\gamma_\mu \left(v_1 + a_1\gamma_5\right) v(p_-) \bar{v}(q_-)\gamma_\mu \left(v_2 + a_2\gamma_5\right) u(q_+), \tag{5}$$

furthermore, v_f and a_f are the vector and axial-vector coupling constants. Introducing the LL, QL, LQ, and QQ structures, correspondingly, we have five structures to which the complete Born amplitude can be reduced: one for the γ exchange amplitude and four for the Z exchange amplitude.

The amplitude of the process $e^+e^- \to f\bar{f}$ in the Standard Model gets in the one-loop approximation contributions from self-energy insertions, vertex corrections, box diagrams, and bremsstrahlung diagrams. We divide these diagrams into several gauge-invariant subsets. We recall that first of all we disentangle a QED subset: QED-vertices and fermionic self-energies, $\gamma\gamma$ and $Z\gamma$ boxes and bremsstrahlung. Then, we divide the remaining diagrams into two more gauge-invariant subsets, giving rise to two improved (or dressed) amplitudes: i) improved γ exchange amplitude with running QED-coupling where only fermion loops contribute, and ii) improved Z exchange amplitude with four, in general complex-valued EW form factors ρ_{ef}, κ_e, κ_f, and κ_{ef}.

The improved Born approximation for the differential in the scattering angle cross section is derived by simple squaring the $(\gamma + Z)$ exchange IBA amplitude $A^{\text{OLA}}(s,t)$ and accounting for proper normalization factors. This differential cross section reads

$$\frac{d\sigma^{\text{IBA}}}{d\cos\vartheta} = \frac{\pi\alpha^2}{s^3}\beta_f N_c \big(\sigma_{\gamma\gamma}^{\text{IBA}} + \sigma_{\gamma Z}^{\text{IBA}} + \sigma_{ZZ}^{\text{IBA}}\big), \tag{6}$$

where $\beta_f = \sqrt{1 - 4m_f^2/s}$ and

$$A^{\rm OLA}(s,t) = ie^2 4I_e^{(3)} I_f^{(3)} \frac{\chi_z(s)}{s} \rho_{ef}(s,t) \times$$
$$\times \Big\{ \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu(1+\gamma_5) - 4|Q_e| s_w^2 \kappa_e(s,t) \gamma_\mu \otimes \gamma_\mu(1+\gamma_5) -$$
$$- 4|Q_f| s_w^2 \kappa_f(s,t) \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu + 16|Q_e Q_f| s_w^4 \kappa_{ef}(s,t) \gamma_\mu \otimes \gamma_\mu \Big\}, \tag{7}$$

here $\chi^{-1}(s) = s - M_Z^2 + is\Gamma_Z/M_Z$.

Form factors are simply related to the one-loop form factors:

$$\begin{aligned} \rho_{ef}(s,t) &= 1 + F_{LL}(s,t) - s_W^2 \Delta_r, \\ \kappa_e(s,t) &= 1 + F_{QL}(s,t) - F_{LL}(s,t), \\ \kappa_f(s,t) &= 1 + F_{LQ}(s,t) - F_{LL}(s,t), \\ \kappa_{ef}(s,t) &= 1 + F_{QQ}(s,t) - F_{LL}(s,t). \end{aligned}$$

Here "1" on the right-hand side is due to the Born amplitude which is included for normalization.

This set of form factors corresponds to an equivalent Born-like expression

$$A^{\rm OLA}(s,t) = i\frac{g^2 e^2}{16\pi^2} 4I_e^{(3)} I_f^{(3)} \frac{\chi_z(s)}{s} \times$$
$$\times \Big\{ \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu(1+\gamma_5) F_{LL}(s,t) - 4|Q_e| s_w^2 \kappa_e(s,t) \gamma_\mu \otimes \gamma_\mu(1+\gamma_5) F_{QL}(s,t) -$$
$$- 4|Q_f| s_w^2 \kappa_f(s,t) \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu F_{LQ}(s,t) - 4|Q_e Q_f| s_w^4 \kappa_{e,f}(s,t) \gamma_\mu \otimes \gamma_\mu F_{QQ}(s,t) \Big\}. \tag{8}$$

Let us highlight two major releases of the ZFITTER program: v.6.21 [11] and v.6.42 [31]. The development of ZFITTER integrates the contributions of numerous theoreticians, whose work is incorporated into the code either as default functions or selectable options through various flags. ZFITTER also uses certain pieces of code from other authors: Fortran function *hadr5* [36], Fortran package *m2tcor* [37, 38], and Fortran package *bkqcdl5 14.f* [39].

The version 6.21 was the main code during the LEP era. Very detailed documentation for it was published in [11]. This release enabled the calculation of realistic observables incorporating complete $\mathcal{O}(\alpha)$ QED and electroweak corrections, along with soft photon exponentiation and some higher-order contributions. This was achieved through several calculation options: a) (improved) Born cross sections; b) a fast mode with application of cuts on s' or combined cuts on acollinearity and the minimum fermion energy $E_{\min}$ for $\sigma_{T,FB}$; c) application of cuts on s' (or on $\xi, E_{\min}$) for the differential distribution $d\sigma/d\cos\vartheta$; d) application of an additional cut on the production angle of antifermions for $\sigma_{T,FB}$. Different interfaces enable fitting experimental data using various sets of free parameters. One should note also that there are two approaches to parameterize the Z boson propagator [40].

Photonic radiative corrections to the e^+e^- annihilation processes, primarily due to radiation from the initial state, are very important numerically, since they can reach several tens of percent. The implementation of these corrections [41–45] to the total and differential cross sections was one of the most involved parts of ZFITTER in terms of length and complexity. Corrections due to the emission of virtual and real fermion pairs also appear to be important, especially those corresponding to the running of the QED coupling constant [46]. The realization of the exponentiation of QED initial–final interference photonic corrections is done according to the procedure developed in [43], based on the general Yennie–Frautschi–Suura theorem [47]. A detailed numerical comparison of the first version of this procedure realization is presented in a review of theoretical calculations for the production of two fermions in electron–positron collisions at LEP2 center-of-mass energies [48]. This review summarizes the outcome of the LEP2 Two-Fermion Working Group, highlighting recent developments and precision improvements essential for accurately modeling these processes and interpreting experimental data collected at the LEP2 collider. The final procedure of the implementation of the initial–final interference itself, as well as the numerical evaluations, are described in detail in the publication accompanying the ZFITTER v.6.42 release [31]. The most important additions of this version are: several higher-order QED corrections to fermion pair production, electroweak one-loop corrections to atomic parity violation, electroweak one-loop corrections to ν_e production, electroweak two-loop corrections to the W boson mass, and corrections to the effective weak mixing angle.

The ZFITTER v.6.44 beta version includes the final results for the $\mathcal{O}(\alpha_s^4)$ QCD corrections to the Z boson and W boson quarkonic partial widths and to the so-called R-ratio by P. Baikov et al. [49].

ZFITTER provides a great flexibility for applications, as well as for comparing with other codes and verifying technical accuracy during program development. It is optionally utilized by other software packages, including SMATASY [50, 51], demonstrating its integration flexibility within various computational tools for electroweak precision analyses.

A comprehensive analysis of the foundational work, a summary of the ZFITTER code development, and a thorough list of the literature are presented in the broader review [13].

ZFITTER and DIZET packages are posted on the SANC homepage [52]. Links to the main publications are also available there.

2.2. Package DIZET

The DIZET package is a component of ZFITTER, it is a library that calculates electroweak, internal mixed QCD⊗EW and QCD⊗QED final-state radiative corrections for several processes within the Standard Model. DIZET can be used in a stand-alone mode and it is often incorporated into other programs such as HECTOR [53], BHAGENE3 [54], KORALZ [55], and KKMC [23].

After the initialization of the DIZET library, the program outputs a set of POs, including the W boson mass, the effective weak mixing angles, the total and partial decay widths of the Z and W bosons, as well as several related electroweak parameters. Subsequently, various DIZET subroutines can be invoked to compute a range of quantities, such as the weak form factors and the running α_s for two-fermion to two-fermion neutral current processes, the weak form factors for two-fermion to two-fermion charged current interactions, the weak form factors for both elastic and deep inelastic neutral current and charged current neutrino–nucleon scattering, and the weak form factors and the running α_s for neutral and charged current deep inelastic scattering processes.

In paper [56], we present the current version of the DIZET package (v. 6.45), describe the updates relative to the previous official release (v. 6.42) and quantify the numerical impact of the newly introduced modifications, controlled by the corresponding options and flags on electroweak pseudo-observables (EWPOs). The contributions added in DIZET v. 6.45 are connected with the completion of the two-loop EW radiative corrections given in [57, 58] which complement earlier works on radiative corrections, namely: the complete fermionic two-loop corrections to the W boson mass [59], the leading $\mathcal{O}(\alpha\alpha_s)$ [60] and next-to-leading $\mathcal{O}(\alpha\alpha_s^2)$ [61–63] QCD corrections, as well as the leading three-loop corrections in an expansion in m_t^2 of the order $\mathcal{O}(\alpha^3)$ and $\mathcal{O}(\alpha^2\alpha_s)$ [64]. These modifications are relevant for future precision HL-LHC studies and the LHC Electroweak Working Group activities.

Recently, the electroweak library GRIFFIN [65] written in C++ was developed, and detailed comparisons with the DIZET library were performed.

2.3. The present status of ZFITTER

The support for updating and improving the ZFITTER code continues up to the present day. One of its most important applications for the elementary particle physics community is available on the webpage of the LEP Electroweak Working Group (LEPEWWG) [12]. The prediction of the Higgs boson mass, assuming the validity of the Standard Model and derived from radiative corrections to precision observables measured at LEP and other experiments, is illustrated in the well-known Blue Band plot (see Subsection 2.4).

2.4. ZFITTER application for the Blue Band

When changing parameters in the realistic observables (ROs) calculation based on POs or SM, theoretical ROs errors are obtained. However, when experimental collaborations analyze their ROs measurements, this procedure is inverted by fitting POs or SM parameters to the measured ROs: when changing the variants of the fitted parameters, they reflect the theoretical uncertainties associated with the ROs calculation. Thus, the theoretical uncertainties for ROs are propagated back to the fitted parameters.

As an example, we provide an estimate of the theoretical uncertainty in the Higgs boson mass based on the parameters of the Standard Model [66]. The Blue Band plot represents the global fit of the Higgs boson mass, integrating precision of experimental measurements with the one of advanced theoretical calculations.

The best constraints on the Higgs boson mass m_H were obtained by fitting all available experimental data. The function $\Delta\chi^2 = \chi^2 - \chi^2_{\text{min}}$ is plotted in Figure 2 as a function of m_H. The minimum of the curve $\Delta\chi^2(m_H)$ determines the best-fit value of the Higgs boson mass, which corresponds to the optimal agreement between the SM predictions and the global set of precision electroweak measurements. As $\Delta\chi^2$ increases when moving away from this point, the quality of the fit worsens, reflecting that such values of m_H are disfavoured by the data.

Thus, from Figure 2 one can directly read off both the preferred value of m_H and the allowed mass range at a given confidence level. The Blue Band indicates the uncertainty from uncalculated higher-order corrections, estimated by both TOPAZ0 and ZFITTER. Taking this uncertainty into account, the 95% confidence level upper limit on m_H is 219 GeV. Also shown is the result (dashed curve) obtained when using $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ of [67].

Restrictions on the known value of the top quark mass and the running coupling constant clearly indicated a value of the Higgs boson mass below 200 GeV. This limitation relied heavily on the validity of the Standard Model. Now we know that the prediction based on the Standard Model was correct.

Discovered experimentally in 2012 at the Large Hadron Collider (LHC), the Higgs boson confirmed the existence of the Higgs field, a pervasive field that interacts with particles to give them mass. The precision electroweak measurements yield a mass of the SM Higgs boson which

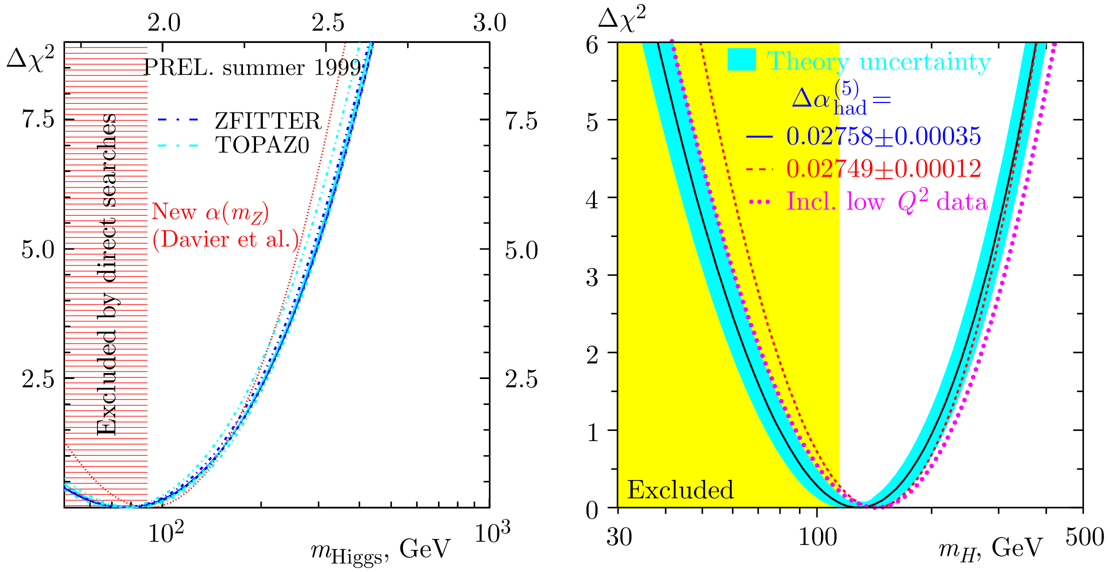


Figure 2. $\Delta\chi^2_{\min}(M^2_H)$ distribution derived from a combined fit of all the world experimental data to the Standard Model exploiting the best knowledge of precision theoretical calculations which is realized in computer codes ZFITTER and TOPAZ0 (left in 1999, right in 2010). It illustrates what we call an indirect discovery of the Higgs boson made via the study of constraints, provided by the precision HEP measurements [68].

is lower than about 152 GeV (one-sided 95% confidence level upper limit derived from $\Delta\chi^2 = 2.7$ for the Blue Band, thus including both the experimental and the theoretical uncertainty).

The plot was created using the version of the ZFITTER package that was current in 2008 (v. 6.43). It is cited in the Scientific Background on the Nobel Prize in Physics 2013: "The Brout–Englert–Higgs-Mechanism, Interactions with Short Range Forces and Scalar Particles" on p. 16.

3. Combined electroweak analysis at LEP/Tevatron

In the LEP era, the analysis of pseudo-observables was carried out within the framework of the Standard Model. The main goal of the theoretical predictions for (pseudo)observables was the testing and verification of the Standard Model [69–71]. It was both a tool and an objective.

Review [35] provides a comprehensive list of definitions for quantities used in analyzing LEP Z resonance results, along with the standard conventions adopted by TOPAZ0 and ZFITTER, and special recommendations from the LEPEWWG. Comparisons encompassed both POs and ROs, such as full cross sections and forward–backward asymmetries, using extrapolated values and realistic cuts applied to obtain complete LEP-based predictions using these codes. Later, paper [48] reports on the developments in the theoretical calculations of the two-fermion production process in the electron–positron collision at LEP2 center-of-mass energies. All available theoretical calculations, performed using Monte Carlo and semi-analytical codes, were collected and applied to obtain predictions for all cross sections, asymmetries, etc., measured in the LEP2 experiments. In the work carried out by several theoretical groups using different methods and approaches, the comparisons of the calculations were provided over a wide range and in the so-called tuned comparisons.

Data from LEP's four experiments yielded about 17 million Z decays, complemented by 600 000 polarized Z decays from the SLD experiment at SLC. Extensive analysis of this data has been done by ALEPH, CDF, D0, DELPHI, L3, OPAL, LEP Electroweak Working Group, Tevatron Electroweak Working Group, SLD Electroweak Working Group, Heavy Flavour Group. They provide analysis of precision electroweak measurements and constraints on the Standard Model by ZFITTER and TOPAZ0. This collaborative effort comparing experimental results with highly precise theoretical calculations, including rigorous tests of renormalization and gauge symmetry preservation, has become one of LEP's cornerstone contributions to the advancement of modern theoretical physics. Throughout the entire LEP program, combined preliminary data on the Z boson parameters from the experiments were systematically collected and analyzed [1, 66, 72–80]. Precision electroweak measurements and SM constraints during the LEP era were detailed in key LEPEWWG reports. These include combinations of Z resonance results like hadronic/leptonic cross sections, forward–backward asymmetries, τ polarization, and heavy flavor widths, often updated for summer conferences. It is also worth noting the papers [68, 81–83] that correspond to post-LEP summaries integrating LEP/SLD data with Tevatron results. In these works, the predictions are calculated with ZFITTER, which incorporate state-of-the-art calculations of radiative corrections in that time.

When analyzing experimental data, the language of pseudo-observables is used. Technically, each LEP experiment extracts pseudo-observables such as M_Z, Γ_Z, σ_h^0, $R_{e,\mu,\tau}$, and $A_{FB}^{0,e,\mu,\tau}$ (see Subsection 4.2 for definitions) from their measured cross sections and asymmetries, which are the realistic observables. As a result, the four LEP experiments adopt a practical model-independent fit strategy: ROs → POs ($\otimes$ Standard Model remainder) for each experiment, followed by averaging the four POs sets while accounting for correlated errors between experiments, to achieve the most precise and statistically correct averaged determination of these fundamental parameters [35]. When applying this procedure, all four LEP experiments have agreed to use the ZFITTER tool because it was providing the most precise, consistent and up-to-date SM calculations with a user-friendly interface. This yields an optimal set of POs values, which constitute important physical quantities in their own right.

Within the SM framework, the ROs can be written in terms of the amplitudes

$$\mathcal{A}_{\rm SM} = \mathcal{A}_\gamma + \mathcal{A}_Z + \mathcal{A}_{\rm nonfact}, \tag{9}$$

where $\mathcal{A}_{\rm nonfact}$ denotes the nonfactorizable contributions that do not factorize in front of a Born-like amplitude, such as weak box diagrams. After computing the matrix element $\mathcal{A}_{\rm SM}$, one squares it, integrates over the phase space to obtain the hard cross section, and then convolutes the result with the QED radiation from the initial and final states and with the QCD final-state radiation.

Higher-order electroweak corrections modify the matrix element structure, requiring complex form factors that depend on the Mandelstam variables and invalidating the separation into the pure γ and Z exchange contributions. Weak box diagrams act as nonresonant insertions in electroweak form factors; at the Z resonance, their one-loop contributions remain small (yielding $\leqslant 10^{-4}$ relative corrections), and neglecting them eliminates t-dependence meanwhile restoring factorization of effective vector and axial-vector couplings. In precision studies of ROs, however, weak boxes are typically retained because the analysis extends off the resonance, up to $s = (M_Z^2 \pm 3)$ GeV. Dropping terms of the order $\mathcal{O}(\alpha\Gamma_Z/M_Z)$ enables full factorization, producing complex effective vector and axial-vector couplings being dependent solely on s, and consistently relating ROs to POs at the required Z pole accuracy. This leads to the Z boson pole approximation, implemented by setting $s = M_Z^2$ in form factors; deconvoluting QED and QCD radiation from measured ROs then establishes a precise ROs-to-POs mapping.

4. Numerical results

4.1. Measurement of τ lepton polarization

As an illustration of a rather advanced ZFITTER's application to experimental data analysis, we present here the τ lepton polarization measurement at LEP [1], where ZFITTER was the primary and sole tool.

The short lifetime of τ lepton allows the measurement of its polarization through the differential distribution of its decay products. At the Born level, the following approximate equality holds for the average τ lepton and Z boson polarization:

$$\langle P_{\tau^-(Z)} \rangle \approx -\frac{2\bar{a}_{\tau(e)}\bar{v}_{\tau(e)}}{\bar{a}^2_{\tau(e)} + \bar{v}^2_{\tau(e)}} \equiv -\mathcal{A}_{\tau(e)}, \tag{10}$$

here $\bar{v}_{\tau(e)}$ and $\bar{a}_{\tau(e)}$ are the vector and axial-vector coupling constants of τ lepton (electron) with the Z boson.

At the Born level, nonzero polarization of Z bosons generates a dependence of $P_\tau(\cos\vartheta)$ on the polar angle between the τ^- momentum direction and the e^- beam direction

$$P_\tau \approx -\frac{\langle P_{\tau^-}\rangle(1+\cos^2\vartheta) + \langle P_Z\rangle 2\cos\vartheta}{(1+\cos^2\vartheta) + \langle P_{\tau^-}\rangle\langle P_Z\rangle 2\cos\vartheta}. \tag{11}$$

Thus, measuring the dependence of τ lepton polarization on the polar angle allows the simultaneous determination of the ratio of coupling constants $\bar{v}/\bar{a}$ for both electrons and leptons. From the lepton universality in the Standard Model, it is possible to determine the electroweak mixing parameter for leptons: $\bar{v}_l/\bar{a}_l = 1 - 4\sin^2\vartheta^{\mathrm{lept}}_{\mathrm{eff}}$.

The final goal of measuring τ lepton polarization at LEP was the precision determination of constants $\mathcal{A}_{\tau,e}$ and refinement of the value of the parameter $\sin^2\vartheta^{\mathrm{lept}}_{\mathrm{eff}}$. For this, both exclusive and inclusive approaches were used.

At the very beginning of the LEP era, the following approach was used to determine the constants A_e and A_τ. At the Born level, the dependence is determined by (10) and the equality is approximate. First, these expressions are held only near the Z pole. When the energy changes from 89 to 93 GeV, the polarization changes by a third of its absolute value. Secondly, even if the accelerator energy exactly corresponds to the Z pole of the resonance, the annihilation processes actually occur at lower energies due to emission of photons from the initial state. Finally, due to loop corrections involving the t quark loops in the vertex $Z\tau\tau$, the difference between P_τ and A_τ depends quite strongly on the mass of the t quark, and similarly, the loop corrections to the vertex Zee lead to uncertainty of the difference between P_Z and A_e. Formula (11) uses approximations that lead to errors in the determination of A_e and A_τ of the order of 0.001−0.002. The real dependence cannot be represented as a simple analytical formula.

In this approach, only $\tau \to e\nu\nu$, $\tau \to \mu\nu\nu$, and $\tau^\pm \to \pi^\pm\pi^\mp\pi^\pm$ decays are identified exclusively. The one-prong hadronic decays $\tau^- \to h^- n\pi^0$ are analyzed inclusively. Here h^- is a charged pion or kaon and n represents the number of neutral pions ($n = 0, 1, \ldots$). The inclusive hadronic decay channel comprises almost 50% of all τ decays. The τ polarization is extracted from inclusive energy spectra of charged and neutral particles, without identifying individual decay modes. The main advantage of this approach is the absence of the potentially largest systematic error due to misidentification of the hadronic decay modes. The method also benefits from a larger statistics which is not reduced by the cuts for the exclusive decay mode selection.

A relatively small disadvantage is the reduction of sensitivity due to the mixing of different decay modes in the inclusive spectra.

For each interval of $\cos\vartheta$, the average polarization of the τ leptons in that range was predicted using ZFITTER, with the values of A_e and A_τ used as input parameters of the programs. The calculation took into account the distribution of the collider's integrated luminosity per collision energy values. The parameters A_e and A_τ were determined by minimum value of the function

$$\chi^2(A_\tau, A_e) = \sum_i \left(\frac{P_i - P_i^{\mathrm{ZF}}(A_\tau, A_e)}{\Delta P_i} \right)^2, \tag{12}$$

where the summation is carried out over all intervals of $\cos\vartheta$, the quantity P_i is the value of polarization measured in the ith interval with experimental uncertainty ΔP_i. P^{ZF} is the average polarization value in the same interval predicted by ZFITTER. For the case of lepton universality, the value of the parameter A_e was determined similarly. This procedure allowed us to extract the parameters of $A_{e,\tau}$, taking into account all known radiative corrections. The value of the parameter A_l, measured in the DELPHI experiment, was $0.1368 \pm 0.0065 \pm 0.0035$ and the value of the effective mixing angle $\sin^2\vartheta_{\mathrm{eff}}^{\mathrm{lept}} = 0.2339 \pm 0.0012$. It should be noted that the results for the effective lepton mixing angle have changed over time, both as a result of the progress in experimental measurements and due to advances in theoretical calculations. Thus, the global fit in 2005 [1] yielded $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} = 0.23160 \pm 0.00017$ and in 2010 the result was reduced to $\sin^2\theta_{\mathrm{eff}}^{\mathrm{lept}} = 0.23138 \pm 0.00013$ [68].

DELPHI has also performed a unique measurement of the τ polarization at LEP2 collision energies between 183 and 209 GeV [84]. The average polarization was found to be $P_\tau = -0.164 \pm 0.117 \pm 0.045$ for the average collision energy 197 GeV. The measurement is in excellent agreement with the ZFITTER prediction.

4.2. Shifts in pseudo-observables

To illustrate the current state of the electroweak pseudo-observables estimation, we present in Table 1 a comparison of the corresponding numerical results obtained by the current versions of ZFITTER and DIZET with those available in one of the publications of the LEP results [35]. The changes in the results are due to both changes in the input parameter set and the inclusion of additional higher-order corrections.

- ZFITTER (LEP) [35]:
  $M_Z = 91.1867$ GeV,
  $M_H = 100$ GeV,
  $m_t = 173.8$ GeV,
  $G_F = 1.16637 \cdot 10^{-5}$ GeV^{-2}.

- Options ZFITTER (2025)
  taken from the PDG Summary Tables [85]:
  $\alpha^{-1}(0) = 137.035999084$,
  $\alpha_s(M_Z) = 0.1180$,
  $M_Z = 91.1880$ GeV,
  $M_H = 125.20$ GeV,
  $m_t = 172.40$ GeV.

Table 1. Values of POs, the first entry is ZFITTER (LEP), the second — ZFITTER (2025), and the third one is relative (δ) or absolute (Δ) difference.

Observable	ZFITTER (LEP)	ZFITTER (2025)	$\delta \cdot 10^3$
M_W, GeV	80.3738	80.3571	0.21
σ_h^0	41.4777	41.4816	0.09
σ_l^0	1.9997	1.9990	0.35
Γ_h, GeV	1.74223	1.74147	0.44
Γ_Z, GeV	2.49573	2.49483	0.36
Γ_ν, MeV	167.234	167.199	0.21
Γ_e, MeV	83.995	83.983	0.14
Γ_μ, MeV	83.995	83.983	0.14
Γ_τ, MeV	83.805	83.793	0.14
Γ_u, MeV	300.154	299.919	0.78
Γ_d, MeV	382.996	382.860	0.36
Γ_c, MeV	300.092	299.854	0.79
Γ_b, MeV	375.993	382.860	17.94
Γ_{inv}, GeV	0.50170	0.50160	0.20
R_l	20.7420	20.7360	0.29
R_b^0	0.215811	0.215833	0.10
R_c^0	0.172246	0.172184	0.36
$\sin^2 \vartheta_{\text{eff}}^{\text{lept}}$	0.231601	0.231507	0.41
$\sin^2 \vartheta_{\text{eff}}^{b}$	0.232950	0.232733	0.93
$\sin^2 \vartheta_{\text{eff}}^{c}$	0.231495	0.231401	0.41
ρ_e	1.00528	1.00503	0.25
ρ_b	0.99424	0.99422	0.02
ρ_c	1.00528	1.00572	0.44

Observable	ZFITTER (LEP)	ZFITTER (2025)	Δ
$A_{FB}^{0,l}$	0.016074	0.016237	0.000163
$A_{FB}^{0,b}$	0.102617	0.103150	0.000533
$A_{FB}^{0,c}$	0.073300	0.073706	0.000406
A_e	0.146396	0.147135	0.000739
A_b	0.934607	0.934746	0.000139
A_c	0.667595	0.667921	0.000326

Below is the list of the POs definitions.

- The total decay width Γ_Z of the Z boson is the sum of all partial decay width: $\Gamma_Z \equiv \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\text{had}} + \Gamma_{\text{inv}}$. Here $\Gamma_{\text{inv}} = N_\nu \Gamma_{\nu\nu}$ is the invisible width from Z decays into neutrinos, where N_ν is the number of light neutrino species.

- The partial decay width $\Gamma_f \equiv 4c_f\Gamma_0 \left(|\mathcal{G}_f^V|^2 R_V^f + |\mathcal{G}_f^A|^2 R_A^f\right) + \Delta_{\text{EW/QCD}}$, where $c_f = 1$ or 3 for leptons or quarks ($f = l, q$), and the radiator factors R_V^f and R_A^f describe the final-state QED and QCD corrections and take into account the fermion mass m_f.

The term of the nonfactorizable corrections is $\Delta_{\mathrm{EW/QCD}} = \Gamma(2)_{\mathrm{EW/QCD}} - \dfrac{\alpha_s}{\pi}\Gamma(1)_{\mathrm{EW}}$.

- The ratios are $R_e \equiv \dfrac{\Gamma_{\mathrm{had}}}{\Gamma_{ee}}$, $R_b^0 \equiv \dfrac{\Gamma_{bb}}{\Gamma_{\mathrm{had}}}$, $R_c^0 \equiv \dfrac{\Gamma_{cc}}{\Gamma_{\mathrm{had}}}$.
- The hadronic and leptonic pole cross sections read $\sigma_h^0 = 12\pi\dfrac{\Gamma_e\Gamma_h}{M_Z^2\Gamma_Z^2}$, $\sigma_l^0 = 12\pi\dfrac{\Gamma_e\Gamma_l}{M_Z^2\Gamma_Z^2}$.
- $A_{FB}^{0f} = \dfrac{3}{4}A_eA_f$, $A_f = \dfrac{2\Re g_Z^f}{(\Re g_Z^f)^2+1}$.

Of course, a lot has changed since the LEP era, and that are not only those updated of the input parameters, but also several new contributions of higher-order radiative corrections have been added into the current version of ZFITTER. Thus, the precise accounting of the mass of the bottom quark led to a significant change in the Γ_b width.

4.3. Dependence on the uncertainty of input parameters

Let us look at the dependence of pseudo-observables on the uncertainty of input parameters. We consider the parametric uncertainties due to variation of the masses m_t, M_H, and M_Z and in addition the dependence on variation of α_s.

The present values of pseudo-observables (EW boson widths and the weak mixing angle) are [85]: $\Gamma_Z = (2495.500 \pm 2.300)$ MeV, $G_Z(\mu\mu) = (83.99 \pm 0.18)$ MeV, $\Gamma_W = (2140 \pm 50)$ MeV, $G_W(l\nu) = (232.4 \pm 1.9)$ MeV, and $\sin^2\vartheta_{\mathrm{eff}} \cdot 10^6 = 231\,610 \pm 40$.

The DIZET program can calculate pseudo-observables and EW form factors in a wide range of input parameters. The first type of theoretical uncertainties are due to variation of the input parameters within experimental errors. Tables 2 and 3 present the dependence of pseudo-observables on the uncertainty of the input parameters: $m_t = 172.40(0.70)$ GeV and $\alpha_s = 0.1180(0.0009)$. As one can see, the parametric uncertainties for the listed pseudo-observables are less than the current experimental errors [85].

Table 2. The effect of the parametric uncertainty in m_t on the magnitudes of POs.

m_t, GeV	172.40−0.70	172.40	172.40+0.70	Diff.
$G_Z(\mu\mu)$, MeV	83.98	83.98	83.99	0.01
Γ_Z, MeV	2494.67	2494.83	2494.99	0.32
$G_W(l\nu)$, MeV	678.83	678.94	679.05	0.22
Γ_W, MeV	2089.55	2089.88	2090.21	0.66
$\sin^2\vartheta_{\mathrm{eff}}^{\mathrm{lept}} \cdot 10^6$	231 528	231 507	231 487	41

Table 3. The effect of the parametric uncertainty in α_s on the magnitudes of POs.

α_s	0.1180−0.0009	0.1179	0.1180+0.0009	Diff.
$G_Z(\mu\mu)$, MeV	83.98	83.98	83.98	0.00
Γ_Z, MeV	2494.35	2494.83	2495.30	0.95
$G_W(l\nu)$, MeV	678.95	678.94	678.92	0.03
Γ_W, MeV	2089.52	2089.88	2090.24	0.72
$\sin^2\vartheta_{\mathrm{eff}}^{\mathrm{lept}} \cdot 10^6$	231 504	231 507	231 510	6

As seen from these tables, the largest uncertainty appears in Γ_W and Γ_Z due to errors in α_s. These parametric uncertainties remain, however, well below the corresponding experimental errors.

In the proposals for release DIZET (version 7.0), we are going to add switches for individual corrections, i.e., vacuum polarization, QED part, etc., as well as to implement new version VP, to add the components to $\Delta\rho$ corrections and to provide the comparison of higher-order corrections with another codes, to implement the discrete tabulation in favour of a direct continuous mapping (to calculate form factors for a given energy directly instead of tabularization and extrapolations).

5. Future ultraprecise studies at e^+e^- colliders

The pursuit of enhanced precision tests of the Standard Model, especially within the Higgs sector, continues to motivate the development of future lepton colliders. Notable proposals such as the International Linear Collider (ILC) [86], the Future Circular e^+e^- Collider (FCC-ee) [87], the Compact Linear Collider (CLIC) [88], and the Chinese Circular Electron Positron Collider (CEPC) [89] aim to provide the experimental precision which is necessary to probe potential deviations from the SM predictions and search for signs of new physics.

These large-scale projects are based on advanced technologies in accelerator and detector techniques. They offer several important advantages that include the potential of much higher luminosities and therefore large data samples. Electron–positron machines are best suited for high-precision studies of the already discovered particles, with obvious advantages due to well-defined kinematics of the initial state and the absence of a large QCD background typical of hadron–hadron colliders. The major challenge in designing a high-energy e^+e^- collider is to overcome the energy loss due to synchrotron radiation emitted by electrons in a circular orbit. One has to build either a linear collider, or a circular collider of a very large radius. The future e^+e^- collider should serve as a powerful Higgs bosons factory, enabling the detection of new and rare processes and allowing measurements of known particle properties with unprecedented precision. New precise measurements will offer high sensitivity to potential subtle deviations from the SM predictions, indicating new physics. Their potential capabilities will allow experimental uncertainties to be reduced by a factor of 10 to 100 compared to previous experiments such as LEP. The status of high-precision calculations for FCC-ee (and other future e^+e^- colliders in general) is described in [90].

For each project, the physics research program is mostly dictated by the energy reach of the given collider. A future hight-energy e^+e^- collider should be essentially a factory of Higgs bosons, operating near the maximum of the Higgsstrahlung process $e^+e^- \to ZH$. At FCC-ee and CEPC, in addition to Higgs physics it is possible to perform a precision measurement of the top quark mass at the top pair production threshold. Other studies in the top quark sector are not foreseen, since both cross section and luminosity are expected to be very small at the $e^+e^- \to tt$ threshold. An important feature of the CEPC and FCC-ee projects is the possibility to collect an enormous statistics of Z bosons (of the order of 10^{12} decays) from running at the 91-GeV peak, i.e., in the so-called Tera-Z mode.

Currently, the Higgs boson mass is known with a relatively low precision of about 240 MeV (combined LHC Run-1 result), to be compared with 2 MeV for the Z boson mass (combined LEP results) or 15 MeV for the W boson (PDG average of LEP and Tevatron results). It is expected that in the future, LHC can reach the precision of 100 MeV or slightly better and it should be considerably improved at a future e^+e^- machine.

In Table 4, we show experimental predictions for EWPOs at FCC-ee as the most stringent among future experimental setups, particularly in the Z resonance region. To account correctly for slight deviations due to loop effects, dedicated programs like DIZET will be of great demand. Table 4 shows the comparison between the estimated FCC-ee experimental precision, the current theoretical uncertainty, and the so-called projected one for representative EWPOs (see Chapter B in [91] and [92] for the exact definitions). By the projected theoretical uncertainty we mean an estimate of the future theoretical uncertainty when the leading three-loop $\mathcal{O}(\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$ corrections will become available.

Table 4. Estimated precision for the direct determination of representative EWPOs at FCC-ee, current theory uncertainties for the SM prediction of these quantities, and the projected theoretical uncertainty [92].

Quantity	FCC-ee	Current theory uncertainty	Projected theory uncertainty
m_W, MeV	0.5−1	4	1
$\sin^2\vartheta_{\text{eff}}^{\text{lept}}$, 10^{-5}	0.6	4.5	1.5
Γ_Z, MeV	0.1	0.4	0.15
R_b, 10^{-5}	6	11	5
R_ℓ, 10^{-3}	1	6	1.5

Further improvements are necessary because theoretical predictions of exclusive Higgs boson decay modes are very sensitive to the Higgs mass used in the calculations. For example, for the $H \to ZZ^*$ channel, the relative uncertainty of the calculated branching fraction is $\Delta BR(H \to ZZ^*)/BR(H \to ZZ^*) \approx 7.7\Delta M_H/M_H$. A 15-MeV uncertainty on M_H is required to reach a permille-level precision on the partial decay width calculations.

Such precision will pose a serious challenge for theoretical physics calculations within the Standard Model and beyond, as the theory must match the exceptional experimental accuracy. This will be accompanied by the development of modern Monte Carlo tools for high-precision simulations of observable quantities in high-energy physics, which can be explored at a future e^+e^- collider.

6. SANC project

After the LEP era, the key shift in theoretical support was moving from semi-analytic codes to Monte Carlo (MC) event generators. The semi-analytic codes still serve as important benchmarks for the inclusive properties of processes. Now the most powerful and influential theoretical support in HEP is provided by Monte Carlo tools. They should generate individual events of particle interactions, with taking into account radiation of additional particles and virtual loop corrections. The generated events are subsequently processed through detector simulation programs and that allows us to perform studies in realistic experimental environment.

After the LEP era, our group began to create modern Monte Carlo tools based on the analytical computation platform we were developing. That is how the SANC project was born. Our experience in creation and support of the ZFITTER program was crucially important in this new project. We now offer several Monte Carlo tools for various processes: integrators MCSANC [93, 94], SANCphot [95], and a generator ReneSANCe [96, 97].

The main goal of the SANC project was the creation of a computer system for semi-automatic calculations of realistic and pseudo-observables for various processes of electroweak interactions “from the SM Lagrangian to event distributions” at the one-loop precision level for experiments at the current and future accelerators. The analytical calculations are implemented at the one-loop level in two gauges in the R_ξ and unitary gauge, and all results are expressed through scalar Passarino–Veltman functions A_0, B_0, C_0, and D_0 [19]. These two equivalent representations allow for not only checking the analytic calculations at the level of analytic expressions, for example, verification of gauge invariance with respect to cancellation of gauge parameters, but also for checking the Ward identities.

The theoretical basement was provided by the book “The Standard Model in the Making: Precision Study of the Electroweak Interactions” [98] written by D.Yu. Bardin and G. Passarino. The key elements of implementation at the global level consist of the project architecture, building block descriptions, procedures, and guidelines. They are described in [99].

At present, several processes are implemented at the one-loop level with leading multi-loop radiative corrections and taking into account longitudinal and transverse polarization for electron–positron annihilation. We have estimated theoretical uncertainties for the polarized annihilation $e^+e^- \to e^+e^-$ [100], $e^+e^- \to \mu^+\mu^-$ [101] , $e^+e^- \to \gamma\gamma$ [102], $e^+e^- \to ZH$ [103, 104], and $e^+e^- \to \gamma Z$ [105], $e^+e^- \to t\bar{t}$ [106] in the same way, i.e., by describing of the one-loop level using the massive helicity approach in the full phase space, see Figure 3.

The target precision for luminosity measurements at future e^+e^- colliders is 10^{-4}. In upcoming studies, our work will focus on improving the treatment of luminosity processes, i.e., Bhabha scattering $e^+e^- \to e^+e^-$, dimuon $e^+e^- \to \mu^+\mu^-$ and diphoton $e^+e^- \to \gamma\gamma$ annihilation processes.

Determining the shape parameters of the Z boson resonance requires precise measurements of the hadron and lepton production cross sections at and near the Z pole. At each FCC-ee interaction point, about $1.5 \cdot 10^{12}$ Z bosons are produced, enabling statistical precision on the order of 10^{-6}. The main sources of limiting uncertainties are the counting of lepton pairs and

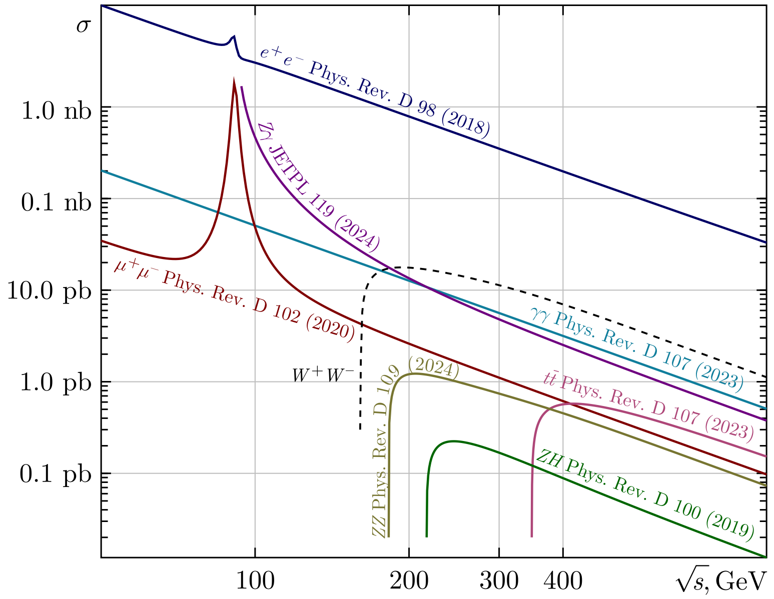


Figure 3. Present status calculations for the various polarized annihilation processes in SANC.

the measurement of the integrated luminosity. Also, the wide-angle diphoton process becomes statistically significant and offers a more reliable determination of the integrated luminosity with respect to systematic uncertainties, compared to the traditional low-angle Bhabha scattering process (main luminosity process in the LEP) [107].

To reduce systematic uncertainties in luminosity and Z boson lineshape measurements in future studies, we plan to incorporate two-loop contributions [108, 109] into the dimuon and diphoton annihilation processes. This development is essential because the FCC-ee requires an extremely precise and stable definition of the geometrical acceptance for the $e^+e^- \to \gamma\gamma$ and $e^+e^- \to l^+l^-$ reactions at the Z pole, with angular cut accuracies at the level of a few microradians, in order to fully exploit the unprecedented statistical sample of Z bosons and to further minimize systematic effects in luminosity and lineshape determinations.

The time has come for ultraprecise photon–photon physics experiments. At the MeV scale, the world's first gamma-photon collider, Genie, focuses on conducting direct experimental tests of light-by-light scattering [110]. It aims to achieve the maximum interaction cross section at a center-of-mass energy of 1–2 MeV, allowing for more precise investigations of this rare process. The photon spectrum is simulated by the CAIN 2.42 [111, 112] program and interfaced to the ReneSANCe generator. Collaboration with the Genie's colleagues enabled us to adapt the generator ReneSANCe for use in this experiment.

7. Discussion

Let us discuss certain advantages and disadvantages of ZFITTER as a project in general and a computer code in particular.

The ZFITTER code is written in the Fortran computer language which is very suitable for coding and numerical evaluation of complicated mathematical formulae. Certain subroutines for special functions, e.g., polylogarithms, and algorithms for numerical integration with reliable control of precision were included in the code. The program itself is rather long, it is separated into several files. Extensive usage of numerous flags and input parameters can be realized only by advanced *educated* users. Nevertheless, an external user of ZFITTER received a complete set of instructions and description of all relevant (actually very numerous) options. Moreover, inside the code itself there are text comments which help to understand the work flow of the code. Advanced users helped a lot in the development of the project by finding bugs in the code and by suggesting improvements and updates. We see that the project benefited greatly from its *open source* strategy, even though the code was not always properly copyrighted.

One of the most valuable features of the code was its reliability and correctness, which was ensured by thorough testing before releasing any new version. Of course, such a long code could not be completely free of errors (bugs), but most of them were minor and were quickly fixed. Another important feature of the project, is the continuous support of the code and of its users. This was especially important during the LEP era, when various groups from experimental collaborations actively used this code for data analysis, fitting pseudo-observables, and estimating uncertainties. The authors of the code promptly replied to requests of the users. This helped improve the code, making it more user-friendly and responsive to real-world needs. Thus, the ZFITTER team's efforts in ongoing code support have been rewarded by the code's development, success, and demand.

As we have discussed, ZFITTER is based on the results of theoretical calculations of many groups besides the team of its authors. Colleagues from experimental collaborations also contributed by ideas on what and how should be evaluated within the code, what should be the input and output parameters.

It is important to note that, practically from the beginning, the project was developed by an international team with participation of researchers from Russia, Germany, and Bulgaria. In the 1980s, Dmitry Bardin and Tord Riemann laid the foundation for a long-term cooperation between JINR and the Zeuthen Institute (now it is DESY-Zeuthen Institute), which became known as the Dubna–Zeuthen Radiative Corrections Group: D. Bardin, A. Akhundov, A. Arbuzov, M. Bilenky, P. Christova, L. Kalinovskaya, T. Riemann, J. Biebel, M. Jack, D. Lehner, A. Leike, S. Riemann, and M. Sachwitz. Tord Riemann was the co-leader of the group and contributed a lot for the success of several common scientific projects, including ZFITTER.

The international composition of the group helped develop connections with various scientific groups from even more countries. Leader of the project D. Bardin worked for a long period at CERN *in situ*. That facilitated the establishment and ongoing maintenance of contacts with experimental collaborations and other competing groups of theorists. Working visits by other members of the ZFITTER team, as well as their participation in international workshops and conferences on high-energy physics, were also highly important for the development of the project.

8. Conclusion

The successes of the experimental measurements performed at the LEP e^+e^- collider in the 1990s posed the challenge of conducting theoretical calculations with a precision comparable to that of experiments ($\approx 10^{-3}$), in order to ensure a high-precision verification of the Standard Model and to determine its parameters that were still unknown at that time. This task was tackled comprehensively by many well-known theoretical groups, including the group from JINR led by D. Bardin, which created the most universal tool, ZFITTER. Eventually, ZFITTER became the gold standard for all LEP experiments and it is still used today for data interpretation within the SM framework.

Moreover, the electroweak correction library DIZET, employed in ZFITTER, serves as the foundation for modern calculations, which are in demand for various types of particles, energy domains, and even for computations of processes beyond the Standard Model. Thus, the work carried out by the international JINR team continues to play an important role in contemporary particle physics at the interface of theory and experiment.

In this way, we described a brief history of the precision theory of electroweak interactions and the contribution of the ZFITTER project to the development of the modern high-energy physics.

What is the role of ZFITTER today? The code is definitely valuable as a benchmark for the SM calculations within the LEP energy range. In particular, it is used for cross checks during the development of new codes [113], for numerical calculation of the parametric dependences of observables [114], for studies of the τ lepton polarization in Z boson decays in proton–proton collisions [115], for justification of the proposal of the FCC-ee and CEPC programs [91, 116]. Future developments of ZFITTER and DIZET include forthcoming version 7.0 releases, which will implement the latest calculations of various higher-order electroweak effects and radiative corrections, in particular those that are presented in [58, 117, 118].

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Conflicts of interest

The authors declare no conflicts of interest.

References

[1] S. Schael et al., Precision electroweak measurements on the Z resonance, Physics Reports 427 (2006) 257–454. `arXiv:hep-ex/0509008`, `doi:10.1016/j.physrep.2005.12.006`.

[2] G. Altarelli, R. Kleiss, C. Verzegnassi (Eds.), Z Physics at LEP-1, Proceedings of the Workshop, Geneva, Switzerland, Sept. 4–5, 1989. Vol. 1: Standard Physics, CERN Yellow Reports: Conference Proceedings, 1989. `doi:10.5170/CERN-1989-008-V-1`.

[3] D. Y. Bardin et al., Electroweak Working Group Report, in: Workshop Group on Precision Calculations for the Z Resonance (2nd meeting held Mar. 31, 3rd meeting held Jun. 13), 1997. `arXiv:hep-ph/9709229`.

[4] W. F. L. Hollik, Radiative corrections in the Standard Model and their role for precision tests of the electroweak theory, Fortschritte der Physik 38 (1990) 165–260. `doi:10.1002/prop.2190380302`.

[5] W. F. L. Hollik, Program WOH, unpublished.

[6] V. Novikov, L. Okun, A. N. Rozanov, M. Vysotsky, LEPTOP, Moscow, ITEP, Mar. 1995. `arXiv:hep-ph/9503308`.

[7] G. Montagna, F. Piccinini, O. Nicrosini, G. Passarino, R. Pittau, TOPAZ0: A program for computing observables and for fitting cross-sections and forward-backward asymmetries around the Z0 peak, Computer Physics Communications 76 (1993) 328–360. `doi:10.1016/0010-4655(93)90060-P`.

[8] G. Montagna, O. Nicrosini, G. Passarino, F. Piccinini, TOPAZ0 2.0: A program for computing deconvoluted and realistic observables around the Z0 peak, Computer Physics Communications 93 (1996) 120–126. `arXiv:hep-ph/9506329`, `doi:10.1016/0010-4655(95)00127-1`.

[9] G. Montagna, O. Nicrosini, F. Piccinini, G. Passarino, TOPAZ0 4.0: A new version of a computer program for evaluation of deconvoluted and realistic observables at LEP-1 and LEP-2, Computer Physics Communications 117 (1999) 278–289. `arXiv:hep-ph/9804211`, `doi:10.1016/S0010-4655(98)00080-0`.

[10] D. Y. Bardin et al., ZFITTER: An analytical program for fermion pair production in e^+e^- annihilation, May 1992. `https://lib-extopc.kek.jp/preprints/PDF/1992/9207/9207126.pdf`, `arXiv:hep-ph/9412201`.

[11] D. Y. Bardin, P. Christova, M. Jack, L. Kalinovskaya, A. Olchevski, S. Riemann, T. Riemann, ZFITTER v.6.21: A semianalytical program for fermion pair production in e^+e^- annihilation, Computer Physics Communications 133 (2001) 229–395. `arXiv:hep-ph/9908433`, `doi:10.1016/S0010-4655(00)00152-1`.

[12] The LEP Electroweak Working Group. `https://lepewwg.web.cern.ch/lepewwg`.

[13] A. Akhundov, A. Arbuzov, S. Riemann, T. Riemann, The ZFITTER project, Physics of Particles and Nuclei 45 (3) (2014) 529–549. `arXiv:1302.1395`, `doi:10.1134/S1063779614030022`.

[14] R. P. Feynman, M. Gell-Mann, Theory of the Fermi interaction, Physical Review 109 (1958) 193–198. `doi:10.1103/PhysRev.109.193`.

[15] E. C. G. Sudarshan, R. E. Marshak, Chirality invariance and the universal Fermi interaction, Physical Review 109 (1958) 1860–1862. `doi:10.1103/PhysRev.109.1860.2`.

[16] L. Michel, Interaction between four half spin particles and the decay of the μ meson, Proceedings of the Physical Society, Section A 63 (1950) 514–531. `doi:10.1088/0370-1298/63/5/311`.

[17] T. Kinoshita, A. Sirlin, Muon decay with parity nonconserving interactions and radiative corrections in the two-component theory, Physical Review 107 (1957) 593–599. `doi:10.1103/PhysRev.107.593`.

[18] T. Kinoshita, A. Sirlin, Radiative corrections to Fermi interactions, Physical Review 113 (1959) 1652–1660. doi:10.1103/PhysRev.113.1652.

[19] G. Passarino, M. J. G. Veltman, One loop corrections for e^+e^- annihilation into $\mu^+\mu^-$ in the Weinberg model, Nuclear Physics B 160 (1979) 151–207. doi:10.1016/0550-3213(79)90234-7.

[20] G. 't Hooft, M. J. G. Veltman, Scalar one loop integrals, Nuclear Physics B 153 (1979) 365–401. doi:10.1016/0550-3213(79)90605-9.

[21] D. Bardin, ZBIZON: A program package for the precision calculation of observables or the process $e^+e^- \to F^+F^-$ around the Z peak, L3 Internal Note 679, Sept. 1989.

[22] D. Y. Bardin, M. S. Bilenky, A. Sazonov, Y. Sedykh, T. Riemann, M. Sachwitz, QED corrections with partial angular integration to fermion pair production in e^+e^- annihilation, Physics Letters B 255 (1991) 290–296. arXiv:hep-ph/9801209, doi:10.1016/0370-2693(91)90250-T.

[23] A. A. Akhundov, D. Y. Bardin, T. Riemann, Hunting the hidden standard Higgs, Physics Letters B 166 (1986) 111–112. doi:10.1016/0370-2693(86)91166-4.

[24] D. Y. Bardin, M. S. Bilenky, A. Chizhov, A. Sazonov, O. Fedorenko, T. Riemann, M. Sachwitz, Analytic approach to the complete set of QED corrections to fermion pair production in e^+e^- annihilation, Nuclear Physics B 351 (1991) 1–48. arXiv:hep-ph/9801208, doi:10.1016/0550-3213(91)90080-H.

[25] P. C. Christova, M. Jack, T. Riemann, Hard photon emission in $e^+e^- \to$ anti-ff with realistic cuts, Physics Letters B 456 (1999) 264–269. arXiv:hep-ph/9902408, doi:10.1016/S0370-2693(99)00528-6.

[26] D. Y. Bardin, M. S. Bilenky, T. Riemann, M. Sachwitz, H. Vogt, DIZET: A program package for the calculation of electroweak one loop corrections for the process $e^+e^- \to f^+f^-$ around the $Z0$ peak, Computer Physics Communications 59 (1990) 303–312. doi:10.1016/0010-4655(90)90179-5.

[27] D. Y. Bardin, P. K. Khristova, O. M. Fedorenko, On the lowest order electroweak corrections to spin 1/2 fermion scattering. 1. The one loop diagrammar, Nuclear Physics B 175 (1980) 435–461. doi:10.1016/0550-3213(80)90021-8.

[28] D. Y. Bardin, O. M. Fedorenko, On high order effects for fermion elastic scattering processes in Weinberg–Salam theory. 1. Renormalization scheme, JINR Preprint P2-11413, Dubna, 1978.

[29] A. A. Akhundov, D. Y. Bardin, T. Riemann, Electroweak one loop corrections to the decay of the neutral vector boson, Nuclear Physics B 276 (1986) 1–13. doi:10.1016/0550-3213(86)90014-3.

[30] D. Y. Bardin, S. Riemann, T. Riemann, Electroweak one loop corrections to the decay of the charged vector boson, Zeitschrift für Physik C 32 (1986) 121–125. doi:10.1007/BF01441360.

[31] A. B. Arbuzov, M. Awramik, M. Czakon, A. Freitas, M. W. Grunewald, K. Monig, S. Riemann, T. Riemann, ZFITTER: A semi-analytical program for fermion pair production in e^+e^- annihilation, from version 6.21 to version 6.42, Computer Physics Communications 174 (2006) 728–758. arXiv:hep-ph/0507146, doi:10.1016/j.cpc.2005.12.009.

[32] A. Sirlin, Radiative corrections in the $SU(2)_L \times U(1)$ theory: A simple renormalization framework, Physical Review D 22 (1980) 971–981. doi:10.1103/PhysRevD.22.971.

[33] D. Y. Bardin, C. Burdik, P. C. Khristova, T. Riemann, Electroweak radiative corrections to deep inelastic scattering at HERA. Neutral current scattering, Zeitschrift für Physik C 42 (1989) 679. doi:10.1007/BF01557676.

[34] D. Y. Bardin, P. K. Khristova, O. M. Fedorenko, On the lowest order electroweak corrections to spin 1/2 fermion scattering. 2. The one loop amplitudes, Nuclear Physics B 197 (1982) 1–44. doi:10.1016/0550-3213(82)90152-3.

[35] D. Y. Bardin, M. Grunewald, G. Passarino, Precision Calculation Project Report, Feb. 1999. arXiv:hep-ph/9902452.

[36] S. Eidelman, F. Jegerlehner, Hadronic contributions to $(g-2)$ of the leptons and to the effective fine structure constant $\alpha(M_Z^2)$, Zeitschrift für Physik C 67 (1995) 585–602. arXiv:hep-ph/

9502298, doi:10.1007/BF01553984.

[37] G. Degrassi, P. Gambino, A. Sirlin, Precise calculation of $M(W)$, $\sin^2\theta(W)(M/Z)$, and $\sin^2\theta_{\text{eff}}^{\text{lept}}$, Physics Letters B 394 (1997) 188–194. arXiv:hep-ph/9611363, doi:10.1016/S0370-2693(96)01677-2.

[38] G. Degrassi, P. Gambino, Two loop heavy top corrections to the $Z0$ boson partial widths, Nuclear Physics B 567 (2000) 3–31. arXiv:hep-ph/9905472, doi:10.1016/S0550-3213(99)00729-4.

[39] B. A. Kniehl, Two loop corrections to the vacuum polarizations in perturbative QCD, Nuclear Physics B 347 (1990) 86–104. doi:10.1016/0550-3213(90)90552-O.

[40] D. Y. Bardin, A. Leike, T. Riemann, M. Sachwitz, Energy dependent width effects in e^+e^- annihilation near the Z boson pole, Physics Letters B 206 (1988) 539–542. doi:10.1016/0370-2693(88)91627-9.

[41] F. A. Berends, W. L. van Neerven, G. J. H. Burgers, Higher order radiative corrections at LEP energies, Nuclear Physics B 297 (1988) 429, [Erratum: Nucl. Phys. B 304 (1988) 921]. doi:10.1016/0550-3213(88)90313-6.

[42] M. Skrzypek, Leading logarithmic calculations of QED corrections at LEP, Acta Physica Polonica B 23 (1992) 135–172.

[43] E. A. Kuraev, V. S. Fadin, On radiative corrections to e^+e^- single photon annihilation at high-energy, Soviet Journal of Nuclear Physics 41 (1985) 466–472.

[44] F. Boudjema et al., Standard Model Processes, in: AGS/RHIC Users Annual Meeting, 1996. arXiv:hep-ph/9601224.

[45] D. Y. Bardin, M. S. Bilenky, W. Beenakker, F. A. Berends, W. L. van Neerven, S. van der Marck, G. Burgers, W. Hollik, T. Riemann, M. Sachwitz, Z Line Shape, in: LEP Physics Workshop, 1989. doi:10.5170/CERN-1989-008-V-1.89.

[46] M. Steinhauser, Leptonic contribution to the effective electromagnetic coupling constant up to three loops, Physics Letters B 429 (1998) 158–161. arXiv:hep-ph/9803313, doi:10.1016/S0370-2693(98)00503-6.

[47] D. R. Yennie, S. C. Frautschi, H. Suura, The infrared divergence phenomena and high-energy processes, Annals of Physics 13 (1961) 379–452. doi:10.1016/0003-4916(61)90151-8.

[48] M. Kobel et al., Two-Fermion Production in Electron–Positron Collisions: Two-Fermion Working Group Report, in: LEP2 Monte Carlo Workshop, 2000. arXiv:hep-ph/0007180, doi:10.5170/CERN-2000-009.269.

[49] P. A. Baikov, K. G. Chetyrkin, J. H. Kuhn, J. Rittinger, Complete $\mathcal{O}(\alpha_s^4)$ QCD corrections to hadronic Z-decays, Physical Review Letters 108 (2012) 222003. arXiv:1201.5804, doi:10.1103/PhysRevLett.108.222003.

[50] A. Leike, T. Riemann, J. Rose, S matrix approach to the Z line shape, Physics Letters B 273 (1991) 513–518. arXiv:hep-ph/9508390, doi:10.1016/0370-2693(91)90307-C.

[51] T. Riemann, Cross-section asymmetries around the Z peak, Physics Letters B 293 (1992) 451–456. arXiv:hep-ph/9506382, doi:10.1016/0370-2693(92)90911-M.

[52] SANC homepage. http://sanc.jinr.ru/zfitter.

[53] A. Arbuzov, D. Y. Bardin, J. Blumlein, L. Kalinovskaya, T. Riemann, HECTOR 1.00: A program for the calculation of QED, QCD and electroweak corrections to ep and $l^\pm N$ deep inelastic neutral and charged current scattering, Computer Physics Communications 94 (1996) 128–184. arXiv:hep-ph/9511434, doi:10.1016/0010-4655(96)00005-7.

[54] J. H. Field, T. Riemann, BHAGENE3: A Monte Carlo event generator for lepton pair production and wide angle Bhabha scattering in e^+e^- collisions near the Z peak, Computer Physics Communications 94 (1996) 53–87. arXiv:hep-ph/9507401, doi:10.1016/0010-4655(95)00131-X.

[55] S. Jadach, B. F. L. Ward, Z. Was, The Monte Carlo program KORALZ, for the lepton or quark pair production at LEP/SLC energies: From version 4.0 to version 4.04, Computer Physics Communications 124 (2000) 233–237. arXiv:hep-ph/9905205, doi:10.1016/S0010-4655(99)

00437-3.

[56] A. Arbuzov, J. Gluza, L. Kalinovskaya, S. Riemann, T. Riemann, V. Yermolchyk, Computer package DIZET v. 6.45, Computer Physics Communications 291 (2023) 108846. arXiv:2301.07168, doi:10.1016/j.cpc.2023.108846.

[57] I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, The two-loop electroweak bosonic corrections to $\sin^2\theta^b_{\text{eff}}$, Physics Letters B 762 (2016) 184–189. arXiv:1607.08375, doi:10.1016/j.physletb.2016.09.012.

[58] I. Dubovyk, A. Freitas, J. Gluza, T. Riemann, J. Usovitsch, Electroweak pseudo-observables and Z-boson form factors at two-loop accuracy, Journal of High Energy Physics 08 (2019) 113. arXiv:1906.08815, doi:10.1007/JHEP08(2019)113.

[59] A. Freitas, W. Hollik, W. Walter, G. Weiglein, Electroweak two loop corrections to the M_W-M_Z mass correlation in the Standard Model, Nuclear Physics B 632 (2002) 189–218, [Erratum: Nucl. Phys. B 666 (2003) 305–307]. arXiv:hep-ph/0202131, doi:10.1016/S0550-3213(02)00243-2.

[60] A. Djouadi, P. Gambino, Electroweak gauge bosons selfenergies: Complete QCD corrections, Physical Review D 49 (1994) 3499–3511, [Erratum: Phys. Rev. D 53 (1996) 4111]. arXiv:hep-ph/9309298, doi:10.1103/PhysRevD.49.3499.

[61] L. Avdeev, J. Fleischer, S. Mikhailov, O. Tarasov, $O(\alpha\alpha_s^2)$ correction to the electroweak ρ parameter, Physics Letters B 336 (1994) 560–566, [Erratum: Phys. Lett. B 349 (1995) 597–598]. arXiv:hep-ph/9406363, doi:10.1016/0370-2693(94)90573-8.

[62] K. G. Chetyrkin, J. H. Kuhn, M. Steinhauser, Corrections of order $\mathcal{O}(G_F M_t^2\alpha_s^2)$ to the ρ parameter, Physics Letters B 351 (1995) 331–338. arXiv:hep-ph/9502291, doi:10.1016/0370-2693(95)00380-4.

[63] K. G. Chetyrkin, J. H. Kuhn, M. Steinhauser, QCD corrections from top quark to relations between electroweak parameters to order α_s^2, Physical Review Letters 75 (1995) 3394–3397. arXiv:hep-ph/9504413, doi:10.1103/PhysRevLett.75.3394.

[64] M. Faisst, J. H. Kuhn, T. Seidensticker, O. Veretin, Three loop top quark contributions to the ρ parameter, Nuclear Physics B 665 (2003) 649–662. arXiv:hep-ph/0302275, doi:10.1016/S0550-3213(03)00450-4.

[65] L. Chen, A. Freitas, GRIFFIN: A C++ library for electroweak radiative corrections in fermion scattering and decay processes, SciPost Physics Codebases 2023 (2023) 18. arXiv:2211.16272, doi:10.21468/SciPostPhysCodeb.18.

[66] R. Barate et al., Search for the Standard Model Higgs boson at LEP, Physics Letters B 565 (2003) 61–75. arXiv:hep-ex/0306033, doi:10.1016/S0370-2693(03)00614-2.

[67] J. F. de Troconiz, F. J. Yndurain, Calculation of $\bar{\alpha}_{\text{QED}}$ on the Z, Physical Review D 65 (2002) 093002. arXiv:hep-ph/0107318, doi:10.1103/PhysRevD.65.093002.

[68] Precision Electroweak Measurements and Constraints on the Standard Model, Dec. 2010. arXiv:1012.2367.

[69] S. L. Glashow, Partial symmetries of weak interactions, Nuclear Physics 22 (1961) 579–588. doi:10.1016/0029-5582(61)90469-2.

[70] S. Weinberg, A model of leptons, Physical Review Letters 19 (1967) 1264–1266. doi:10.1103/PhysRevLett.19.1264.

[71] A. Salam, Weak and electromagnetic interactions, Conference Proceedings C 680519 (1968) 367–377. doi:10.1142/9789812795915_0034.

[72] D. Albaneo, Combined Preliminary Data on Z Parameters from the LEP Experiments and Constraints on the Standard Model, in: 27th International Conference on High-Energy Physics, 1994.

[73] A Combination of Preliminary LEP Electroweak Measurements and Constraints on the Standard Model, in: 17th International Symposium on Lepton Photon Interactions, 1995.

[74] D. Abbaneo et al., A Combination of Preliminary Electroweak Measurements and Constraints

on the Standard Model, Jan. 2000.

[75] A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model, Feb. 2001. `arXiv:hep-ex/0103048`.

[76] D. Abbaneo et al., A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model, Dec. 2001. `arXiv:hep-ex/0112021`.

[77] A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model, Dec. 2003. `arXiv:hep-ex/0312023`.

[78] A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model, Dec. 2004. `arXiv:hep-ex/0412015`.

[79] A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model, Nov. 2005. `arXiv:hep-ex/0511027`.

[80] J. Alcaraz et al., A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model, Dec. 2006. `arXiv:hep-ex/0612034`.

[81] J. Alcaraz et al., Precision Electroweak Measurements and Constraints on the Standard Model, Dec. 2007. `arXiv:0712.0929`.

[82] Precision Electroweak Measurements and Constraints on the Standard Model, Nov. 2008. `arXiv:0811.4682`.

[83] Precision Electroweak Measurements and Constraints on the Standard Model, Nov. 2009. `arXiv:0911.2604`.

[84] J. Abdallah et al., Measurement of the tau lepton polarisation at LEP2, Physics Letters B 659 (2008) 65–73. `arXiv:0710.1368`, `doi:10.1016/j.physletb.2007.10.022`.

[85] S. Navas et al., Review of particle physics, Physical Review D 110 (3) (2024) 030001. `doi:10.1103/PhysRevD.110.030001`.

[86] H. Baer et al., The International Linear Collider Technical Design Report: Vol. 2 — Physics, Jun. 2013. `arXiv:1306.6352`.

[87] A. Abada et al., FCC-ee: The lepton collider: Future Circular Collider Conceptual Design Report, Vol. 2, European Physical Journal: Special Topics 228 (2) (2019) 261–623. `doi:10.1140/epjst/e2019-900045-4`.

[88] Physics and Detectors at CLIC: CLIC Conceptual Design Report, Feb. 2012. `arXiv:1202.5940`, `doi:10.5170/CERN-2012-003`.

[89] M. Dong et al., CEPC Conceptual Design Report: Vol. 2 — Physics and Detector, Nov. 2018. `arXiv:1811.10545`.

[90] A. Blondel, J. Gluza, S. Jadach, P. Janot, T. Riemann (Eds.), Theory for the FCC-ee: Report on the 11th FCC-ee Workshop: Theory and Experiments, Vol. 3/2020 of CERN Yellow Reports: Monographs, CERN, Geneva, 2019. `arXiv:1905.05078`, `doi:10.23731/CYRM-2020-003`.

[91] A. Blondel et al., Standard Model Theory for the FCC-ee Tera-Z Stage, in: Mini Workshop on Precision EW and QCD Calculations for the FCC Studies: Methods and Techniques, Vol. 3/2019 of CERN Yellow Reports: Monographs, CERN, Geneva, 2018. `arXiv:1809.01830`, `doi:10.23731/CYRM-2019-003`.

[92] A. Freitas et al., Theoretical Uncertainties for Electroweak and Higgs-Boson Precision Measurements at FCC-ee, Jun. 2019. `arXiv:1906.05379`.

[93] S. G. Bondarenko, A. A. Sapronov, NLO EW and QCD proton–proton cross section calculations with MCSANC-v1.01, Computer Physics Communications 184 (2013) 2343–2350. `arXiv:1301.3687`, `doi:10.1016/j.cpc.2013.05.010`.

[94] A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, U. Klein, V. Kolesnikov, L. Rumyantsev, R. Sadykov, A. Sapronov, Update of the MCSANC Monte Carlo integrator, v. 1.20, Journal of Experimental and Theoretical Physics Letters 103 (2) (2016) 131–136. `arXiv:1509.03052`, `doi:10.1134/S0021364016020041`.

[95] S. G. Bondarenko, L. V. Kalinovskaya, A. A. Sapronov, Monte-Carlo tool SANCphot for polarized

$\gamma\gamma$ collision simulation, Computer Physics Communications 294 (2024) 108929. arXiv:2201.04350, doi:10.1016/j.cpc.2023.108929.

[96] R. Sadykov, V. Yermolchyk, Polarized NLO EW e^+e^- cross section calculations with ReneSANCe-v1.0.0, Computer Physics Communications 256 (2020) 107445. arXiv:2001.10755, doi:10.1016/j.cpc.2020.107445.

[97] S. Bondarenko, Y. Dydyshka, L. Kalinovskaya, R. Sadykov, V. Yermolchyk, Hadron–hadron collision mode in ReneSANCe-v1.3.0, Computer Physics Communications 285 (2023) 108646. arXiv:2207.04332, doi:10.1016/j.cpc.2022.108646.

[98] D. Yu. Bardin, G. Passarino, The Standard Model in the Making: Precision Study of the Electroweak Interactions, Clarendon Press, Oxford, 1999.

[99] A. Andonov, A. Arbuzov, D. Bardin, S. Bondarenko, P. Christova, L. Kalinovskaya, G. Nanava, W. von Schlippe, SANCscope — v.1.00, Computer Physics Communications 174 (2006) 481–517, [Erratum: Comput. Phys. Commun. 177 (2007) 623–624]. arXiv:hep-ph/0411186, doi:10.1016/j.cpc.2005.12.006.

[100] D. Bardin, Y. Dydyshka, L. Kalinovskaya, L. Rumyantsev, A. Arbuzov, R. Sadykov, S. Bondarenko, One-loop electroweak radiative corrections to polarized Bhabha scattering, Physical Review D 98 (1) (2018) 013001. arXiv:1801.00125, doi:10.1103/PhysRevD.98.013001.

[101] S. Bondarenko, Y. Dydyshka, L. Kalinovskaya, R. Sadykov, V. Yermolchyk, One-loop electroweak radiative corrections to lepton pair production in polarized electron–positron collisions, Physical Review D 102 (3) (2020) 033004. arXiv:2005.04748, doi:10.1103/PhysRevD.102.033004.

[102] S. Bondarenko, Y. Dydyshka, L. Kalinovskaya, A. Kampf, L. Rumyantsev, R. Sadykov, V. Yermolchyk, One-loop radiative corrections to photon-pair production in polarized positron–electron annihilation, Physical Review D 107 (7) (2023) 073003. arXiv:2211.11467, doi:10.1103/PhysRevD.107.073003.

[103] S. Bondarenko, Y. Dydyshka, L. Kalinovskaya, L. Rumyantsev, R. Sadykov, V. Yermolchyk, One-loop electroweak radiative corrections to polarized $e^+e^- \to ZH$, Physical Review D 100 (7) (2019) 073002. arXiv:1812.10965, doi:10.1103/PhysRevD.100.073002.

[104] A. Arbuzov, S. Bondarenko, L. Kalinovskaya, R. Sadykov, V. Yermolchyk, Electroweak effects in $e^+e^- \to ZH$ process, Symmetry 13 (7) (2021) 1256. doi:10.3390/sym13071256.

[105] S. G. Bondarenko, E. V. Dydyshka, L. V. Kalinovskaya, L. A. Rumyantsev, R. R. Sadykov, V. L. Ermol'chik, One-loop electroweak radiative corrections to polarized $e^+e^- \to \gamma Z$ process, Pis'ma v Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki 119 (2) (2024) 75–81.

[106] A. Arbuzov, S. Bondarenko, L. Kalinovskaya, R. Sadykov, V. Yermolchyk, Electroweak radiative corrections to polarized top quark pair production, Physical Review D 107 (11) (2023) 113006. arXiv:2305.09569, doi:10.1103/PhysRevD.107.113006.

[107] M. Benedikt et al., Future Circular Collider Feasibility Study Report: Vol. 1 — Physics, Experiments, Detectors, Apr. 2025. arXiv:2505.00272, doi:10.17181/CERN.9DKX.TDH9.

[108] R. E. Gerasimov, P. A. Krachkov, R. N. Lee, Electron–positron annihilation into heavy leptons at two loops, Journal of High Energy Physics 08 (2025) 118. arXiv:2503.09245, doi:10.1007/JHEP08(2025)118.

[109] R. N. Lee, V. A. Stotsky, Master integrals for $e^+e^- \to 2\gamma$ process at large energies and angles, Journal of High Energy Physics 12 (2024) 106. arXiv:2410.03336, doi:10.1007/JHEP12(2024)106.

[110] T. Takahashi et al., Light-by-light scattering in a photon–photon collider, The European Physical Journal C 78 (11) (2018) 893, [Erratum: Eur. Phys. J. C 82 (2022) 404]. arXiv:1807.00101, doi:10.1140/epjc/s10052-018-6364-1.

[111] P. Chen, G. Horton-Smith, T. Ohgaki, A. W. Weidemann, K. Yokoya, CAIN: Conglomerat d'ABEL et d'interactions nonlineaires, Nuclear Instruments and Methods in Physics Research, Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 355 (1995) 107–

110. doi:10.1016/0168-9002(94)01186-9.

[112] K. Yokoya, User Manual of CAIN, Version 2.42, User Manual of CAIN, Version 2.40, 2018. doi:{http://ilc.kek.jp/yokoya/CAIN}.

[113] M. Ciuchini, E. Franco, S. Mishima, L. Silvestrini, Electroweak precision observables, new physics and the nature of a 126 GeV Higgs boson, Journal of High Energy Physics 08 (2013) 106. arXiv:1306.4644, doi:10.1007/JHEP08(2013)106.

[114] J. Ellis, T. You, Sensitivities of prospective future e^+e^- colliders to decoupled new physics, Journal of High Energy Physics 03 (2016) 089. arXiv:1510.04561, doi:10.1007/JHEP03(2016)089.

[115] A. Hayrapetyan et al. (CMS Collab.), Measurement of the τ lepton polarization in Z boson decays in proton–proton collisions at $\sqrt{s} = 13$ TeV, Journal of High Energy Physics 01 (2024) 101. arXiv:2309.12408, doi:10.1007/JHEP01(2024)101.

[116] Z. Zhao, S. Yang, M. Ruan, M. Liu, L. Han, Measurement of the effective weak mixing angle at the CEPC*, Chinese Physics C 47 (12) (2023) 123002. arXiv:2204.09921, doi:10.1088/1674-1137/acf91f.

[117] J. Blümlein, A. De Freitas, C. G. Raab, K. Schönwald, The $O(\alpha^2)$ initial state QED corrections to e^+e^- annihilation to a neutral vector boson revisited, Physics Letters B 791 (2019) 206–209. arXiv:1901.08018, doi:10.1016/j.physletb.2019.02.038.

[118] A. B. Arbuzov, U. E. Voznaya, Higher-order NLO initial state QED radiative corrections to e^+e^- annihilation revisited, Physical Review D 109 (11) (2024) 113002. arXiv:2405.03443, doi:10.1103/PhysRevD.109.113002.