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Nucleus scattering on ^{12}C and ^{27}Al targets in the complete Glauber theory

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Abstract

The straightforward calculations of the nucleus–nucleus scattering cross sections are carried out in Glauber approach using the generating function method. It allows for the resummation of all orders of Glauber theory. The results are obtained for a number of light nuclei isotopes scattering on ^{12}C and ^{27}Al targets. Their radii, extracted by comparing the calculated cross sections with the experimental ones, are presented.

Keywords: Glauber theory, reaction cross section, interaction cross section, halo nuclei

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1. Introduction

The information on various aspects of the nuclear structure comes mainly from the experimental data on the collision of a beam nucleus A with a target B. The proton and neutron distributions in the nucleus are probed by comparing the data from electron and proton elastic scattering. The only way to study the density distribution of unstable nuclei, in particular, of halo nuclei, is to scatter them off a target. There are technical difficulties to take hydrogen as a target, so targets are mainly made of stable nuclei such as ^{12}C or ^{27}Al. The analysis of these experiments at the energies higher than several hundred MeV per nucleon is typically performed in the Glauber theory. The complexity of calculations grows rapidly when going from proton–nucleus to nucleus–nucleus scattering because of high-order corrections that have to be accounted for. The method to sum them up by using the generation function has been proposed in [1].

What is directly measured in the nuclear reactions is the interaction cross section. It is defined as the cross section of the process when the beam nucleus A scatters without being excited or disintegrated (and is registered by a spectrometer) whereas it is allowed for the target nucleus B,

$$\sigma^I_{AB} = \sigma^{\rm tot}_{AB} - \sigma^{\rm el}_{AB} - \sigma_{AB\to AB^*} = \sigma^{\rm tot}_{AB} - \sigma_{AB\to AB'}.$$

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Here B^* stands for all excited or disintegrated states of the target nucleus. Introducing the reaction, or the total inelastic, cross section

$$\sigma^R_{AB} = \sigma^{\rm tot}_{AB} - \sigma^{\rm el}_{AB},$$

it can be rewritten as

$$\sigma^I_{AB} = \sigma^R_{AB} - \sigma_{AB\to AB^*}.$$

It is the interaction cross section that is really measured because of the experimental difficulty to distinguish a pure elastic scattering from processes giving rise to the target excitation or disintegration. The beam energy loss in the latter case is very small compared to the initial value to detect. It is important to note that apart from the excited or disintegrated states of the beam and target nuclei the total cross section gets contributions from the production of extra particles, mainly π mesons. They are not detected as well as the final state of the target.

Usually the difference between σ^R_{AB} and σ^I_{AB} is assumed to be negligible. The Monte-Carlo simulation results into the value 2–3% of σ^R_{AB} [2]. In this paper we present a complete analytical Glauber calculation of the reaction and the interaction cross sections for several relatively light nuclei.

It was experimentally found [3, 4] that the radii of some isotopes are abnormally large. The most evident case is ^{11}Li nucleus. It has been proposed to treat it as a compact core ^{9}Li surrounded with two neutron halo whose radius is large. There are other nuclei for which the similar core plus halo structure is assumed.

The radii of unstable nuclei can be extracted from their scattering on a target. To this end, we calculate the cross section of a given nucleus scattering on ^{12}C and ^{27}Al targets taking both densities in the simple gaussian form. Comparing the results with the experimental value and assuming it to be precisely the interaction cross section we extract the mean square radius. To estimate the difference between the two cross sections, we calculate with the obtained radius the reaction cross section.

Clearly, the simple Gauss parameterization does not incorporate shell effects that could be important even for light nuclei. Woods–Saxon parameterization seems to be favored when passing to more heavy nuclei such as ^{27}Al. However, there is only one experimental quantity available for each nucleus, namely, its cross section. That is why we have only one density parameter to fit. On the other hand, our main goal is to estimate the difference between two types of cross sections. From this point of view, there is not much sense to chose more advanced parameterizations.

2. Cross sections in the Glauber approach

The amplitude of the elastic scattering of the incident nucleus A on the fixed target nucleus B reads in the Glauber theory

$$f_{AB}(q) = \frac{ip}{2\pi}\int d^2b e^{iqb}\big[1 - s_{AB}(b)\big]. \tag{1}$$

Here p is a relative momentum in the central of mass frame, q is the transferred momentum. The impact parameter b is a two-dimensional vector in the transverse plane with respect to relative momentum of the colliding nuclei A, B. The evaluation of the function $s_{AB}(b)$ relies on the short range of the strong interaction. Due to this property, the phase shift on a nucleus comes out the sum of those for the independent scattering of the constituent nucleons. The function $s_{AB}(b)$ then reads

$$s_{AB}(b) = \langle A, |\langle B| \left\{ \prod_{ij} \left[1 - \Gamma_{NN}(b + x_i - y_j)\right] \right\} |A, \rangle B\rangle, \tag{2}$$

where

$$\Gamma_{NN}(b) \equiv 1 - s_{NN}(b) = \frac{1}{2\pi i p} \int d^2 q e^{iqb} f_{NN}(q),$$

$f_{NN}(q)$ is the nucleon–nucleon elastic scattering amplitude related to the phase shift $\chi_{NN}(b)$ as $s_{NN}(b) = e^{i\chi_{NN}(b)}$. The brackets stand for an average over the nucleons' positions x_i and y_j lying in the same plane with the impact parameter. Each pair $\{i, j\}$ enters the product only once, meaning that each nucleon from the projectile nucleus can scatter on each nucleon from the target no more than once.

The standard parameterization of the elastic nucleon–nucleon amplitude is

$$f_{NN}(q) = ip\frac{\sigma_{NN}^{\rm tot}}{4\pi} e^{-\frac{1}{2}\beta q^2}, \tag{3}$$

where $\sigma_{NN}^{\rm tot}$ is the total nucleon–nucleon cross section. The slope β is related to an effective interaction radius $a^2 = 2\pi\beta$. The parameterization (3) is purely imaginary. The ratio of the real to the imaginary part of the nucleon–nucleon scattering amplitude at the projectile energy 1 GeV is -0.275 [5]. Only even powers of this ratio enter the cross section, so we neglect hereafter the real part of the amplitude.

The elastic amplitude is simple related to the total cross section through the optic theorem,

$$\sigma_{AB}^{\rm tot} = \frac{4\pi}{p} {\rm Im} f_{AB}(q = 0) = 2 \int d^2 b \left[1 - s_{AB}(b)\right].$$

The difference between the total cross section and the integrated elastic cross section,

$$\sigma_{AB}^{\rm el} = \int d^2 b \left[1 - s_{AB}(b)\right]^2, \tag{4}$$

yields the reaction cross section,

$$\sigma_{AB}^{r} = \sigma_{AB}^{\rm tot} - \sigma_{AB}^{\rm el} = \int d^2 b \left[1 - s_{AB}^2(b)\right].$$

Recall that the above cross sections are written assuming the elastic amplitude to be purely imaginary.

The amplitude of the process $A + B \to A + B^*$ when the target nucleus B turns into an excited or a disentegrated state B^* takes a form similar to (2),

$$s_{AB^*}(b) = \langle A, B^*| \left\{ \prod_{ij} \left[1 - \Gamma_{NN}(b + x_i - y_j)\right] \right\} |A, B\rangle. \tag{5}$$

Strictly speaking, this formula goes a bit beyond the standard Glauber approach which deals with an *elastic* scattering of A and B. However, the main assumption it relies on is the same — a composite objects' scattering is expressed through the purely elastic re-scatterings of their constituents. That is why the extension (5) seems to be quite reasonable.

Denoting through $|B'\rangle = \{|B\rangle, |B^*\rangle\}$, the complete set of all target states,

$$\sum_{B'} |B'\rangle\langle B'| = |B\rangle\langle B| + \sum_{B^*} |B^*\rangle\langle B^*| = 1, \tag{6}$$

one gets for the cross section $AB \to AB'$:

$$\sigma_{AB\to AB'} = \int d^2b\big[1 - 2s_{AB}(b) + J_{AB}(b)\big], \tag{7}$$

where

$$J_{AB}(b) = \sum_{B'} \langle A|\langle B| \left\{ \prod_{ij} \big[1 - \Gamma_{NN}(b + x_i - x'_j)\big] \right\} |A\rangle|B'\rangle \times \\ \times \langle A|\langle B'| \left\{ \prod_{i'j'} \big[1 - \Gamma_{NN}(b + y_i - y'_j)\big] \right\} |B\rangle|A\rangle, \tag{8}$$

with the pairs $\{i, j\}$ and $\{i', j'\}$ again entering each products no more than once.

Subtracting from (7) the elastic cross section (4), we arrive at the interaction cross section,

$$\sigma^I_{AB} = \sigma^r_{AB} - \sigma_{AB\to AB^*} = \sigma^{\rm tot}_{AB} - \sigma_{AB\to AB'} = \int d^2b\big[1 - J_{AB}(b)\big]. \tag{9}$$

The main obstacle to deal with the Glauber amplitudes (2) or (8) is complicated combinatorial structure they have for the scattering of nuclei. Expanding the amplitude in the number of nucleon–nucleon scatterings Γ_{NN} gives rise to a kind of diagram technique. As the complexity of various terms increases, additional simplifications are commonly used to obtain an analytical expression. The most popular is the so-called optical approximation that sums up the contributions with no more than one scattering for each nucleon. In other words, only the terms with different indices i, j are kept in the product (2), see, e.g., [6, 7]. One can mention as well the rigid target approximation taking into account extra type of diagrams [8].

The generating function formalism developed in [1] allows to get the nuclear scattering amplitudes in all orders of Glauber theory. Below we apply it to evaluate the reaction and the interaction cross sections.

3. Generating function

Firstly, we rewrite the amplitudes (2) and (5) through nucleon distributions in the colliding nuclei. Supposing them to be the product of one-nucleon densities,

$$\rho_N(x_1, \ldots, x_N) = \prod_{i=1}^{N} \rho(r_1)\cdots\rho(r_N), \qquad \int d^3r\rho(r) = 1, \tag{10}$$

we get

$$S_{AB}(b) = \int \prod_{i=1}^{A} d^2x_i \int \prod_{j=1}^{B} d^2y_j \rho^{\perp}_A(x_i - b)\rho^{\perp}_B(y_j) \left\{ \prod_{ij} \big[1 - \Gamma_{NN}(x_i - y_j)\big] \right\}, \tag{11}$$

where the transverse nucleon densities are determined through three-dimensional ones integrated over longitudinal coordinates,

$$\rho^{\perp}_{A,B}(x) = \int dz \rho_{A,B}(z,x), \quad \int d^2x \rho^{\perp}_{A,B}(x) = 1.$$

The generating function method [1] relies on the identity

$$C_0 \int D\Phi D\Phi^* \exp\Big\{ -\int d^2x d^2y \Phi(x) \Delta^{-1}(x-y) \Phi^*(y) + \sum_i \Phi(x_i) + \sum_j \Phi^*(y_j) \Big\} =$$
$$= \exp\Big\{ \sum_{i,j} \Delta(x_i - y_j) \Big\} = \Big\{ \prod_{ij} [1 - \Gamma_{NN}(x_i - y_j)] \Big\}. \quad (12)$$

The functional integral here is understood as an infinite product of two dimensional integrals over all x points, $D\Phi D\Phi^* = \prod_x 2i d\mathrm{Re}\Phi(x) d\mathrm{Im}\Phi(x)$. The formula (12) is a peculiar case of the general Gaussian integral

$$C_0 \int D\Phi D\Phi^* \exp\Big\{ -\int d^2x d^2y \Phi(x) \Delta^{-1}(x-y) \Phi^*(y) +$$
$$+ \int d^2y J(y) \Phi^*(y) + \int d^2x J^*(x) \Phi(x) \Big\} = \exp\Big\{ \int d^2x d^2y J(x) \Delta(x-y) J^*(y) \Big\},$$

for the external sources of the form

$$J(x) = \sum_i \delta^{(2)}(x - x_i), \quad J^*(y) = \sum_j \delta^{(2)}(y - y_j).$$

The inverse Δ^{-1} is understood in operator sense, $\int d^2z \Delta(x-z) \Delta^{-1}(z-y) = \delta^{(2)}(x-y)$. If the function $\Delta(x-y)$ is chosen to obey the equation

$$e^{\Delta(x-y)} - 1 = -\Gamma_{NN}(x-y), \quad (13)$$

one gets the last term in the expression (12). The normalization constant C_0 being proportional to the functional determinant $\mathrm{Det}\Delta$ does not depend on the positions x_i, y_j and drops out at the end (from the formula (16) below). Combining the formulas (2) and (12) one gets

$$S_{AB}(b) = C_0 \int \frac{D\Phi D\Phi^*}{2\pi i} \exp\left\{ -\int d^2x d^2y \Phi(x) \Delta^{-1}(x-y) \Phi^*(y) \right\} \times$$
$$\times \left[\int d^2x \rho^{\perp}_A(x-b) e^{\Phi(x)} \right]^A \left[\int d^2y \rho^{\perp}_B(y) e^{\Phi^*(y)} \right]^B. \quad (14)$$

An efficient way to deal with the integral (14) is through the generating function,

$$Z(u,v) = \int \frac{D\Phi D\Phi^*}{2\pi i} \exp\left\{ -\int d^2x d^2y \Phi(x) \Delta^{-1}(x-y) \Phi^*(y) \right.$$
$$\left. + u \int d^2x \rho^{\perp}_A(x-b) e^{\Phi(x)} + v \int d^2x \rho^{\perp}_B(x) e^{\Phi^*(x)} \right\}, \quad (15)$$

$$S_{AB}(b) = \frac{1}{Z(0,0)} \frac{\partial^A}{\partial u^A} \frac{\partial^B}{\partial v^B} Z(u,v) \Big|_{u=v=0}. \quad (16)$$

To go further we work out short distance nature of the nuclear forces. The nucleon–nucleon scattering amplitude (3) reads in the coordinate representation

$$\Gamma_{NN}(x) = \frac{\sigma_{NN}^{\rm tot}}{4\pi\beta} e^{-\frac{x^2}{2\beta}}, \tag{17}$$

the value $a = \sqrt{2\pi\beta}$ being of the order of the interaction radius. Assuming a to be small at the nuclear scale, the amplitude can be treated as a point-like function,

$$\Gamma_{NN}(x) \simeq \frac{1}{2}\sigma_{NN}^{\rm tot}\delta^{(2)}(x). \tag{18}$$

Point-like $\Delta(x-y)$ makes the $\Phi(x)$ integrals (15) to be independent for different x values. It turns the functional integral into an infinite product of the finite dimension integrals, separately evaluated for each x.

The discrete version of the integral (12) is derived from the sequence of identities. The first is

$$\int \frac{D\Phi D\Phi^*}{2\pi i} e^{-\Phi\Phi^*} \frac{(\Phi^*)^m}{\sqrt{m!}} \frac{(\Phi)^n}{\sqrt{n!}} = \delta_{m,n}, \tag{19}$$

which is easily checked in the polar coordinates, then

$$\int \frac{D\Phi D\Phi^*}{2\pi i} e^{-\Phi\Phi^*} e^{a\Phi^*} e^{b\Phi} = \sum_{m,n} \int \frac{D\Phi D\Phi^*}{2\pi i} e^{-\Phi\Phi^*} \frac{a^m}{\sqrt{m!}} \frac{b^n}{\sqrt{n!}} \frac{(\Phi^*)^m}{\sqrt{m!}} \frac{(\Phi)^n}{\sqrt{n!}} = e^{ab} \tag{20}$$

and

$$\int \prod_k \frac{d\Phi_k d\Phi_k^*}{2\pi i} e^{-\frac{1}{y}\sum_k \Phi_k\Phi_k^*} e^{\sum_k (a_k\Phi_k^* + b_k\Phi_k)} = e^{y\sum_k a_k b_k}, \tag{21}$$

which gives for the (12) discreet counterpart

$$\prod_{x_n} \int \frac{d\Phi(x_n) d\Phi^*(x_n)}{2\pi i} \exp\left\{-\sum_n \frac{1}{y}\Phi(x_n)\Phi^*(x_n) + \sum_i \Phi(x_i) + \sum_j \Phi^*(y_j)\right\} = \\ = \exp\big\{y\sum_{i,j}\delta_{x_i,y_j}\big\}, \tag{22}$$

where δ_{x_i,y_j} is Kronecker symbol for discrete nucleons' coordinates, $\delta_{x_i,y_j}/a^2 \to \delta^{(2)}(x_i - y_j)$ for $a \to 0$. Since

$$e^{y\delta_{x_i,y_j}} = 1 + (e^y - 1)\delta_{x_i,y_j},$$

the right-hand side of the identity (22) yields

$$\prod_{i,j}\left[1 - \frac{1}{2}\frac{\sigma_{NN}^{\rm tot}}{a^2}\delta_{x_i,y_j}\right] \to \prod_{i,j}\left[1 - \Gamma_{NN}(x_i - y_j)\right],$$

whereas the equation (13) translates into

$$e^y - 1 = g\frac{1}{2}\frac{\sigma_{NN}^{\rm tot}}{a^2}. \tag{23}$$

Referring back to the formula (15) and replacing in the discreet version $\int d^2x \to a^2 \sum_{x_k}$, one gets the integral

$$Z(u,v) = \int \prod_{x_k} \frac{d\Phi(x_k)d\Phi^*(x_k)}{2\pi i} e^{-\frac{1}{y}\Phi(x_k)\Phi^*(x_k)} \exp\big\{ua^2\rho_A^\perp(x_k-b)e^{\Phi(x_k)} + v\rho_B^\perp(x_k)e^{\Phi^*(x_k)}\big\}. \quad (24)$$

With the help of the equality derived from Eq. (20)

$$\int \frac{d\Phi d\Phi^*}{2\pi i} e^{-\frac{1}{y}\Phi\Phi^*} \exp\big\{ue^{\Phi} + ve^{\Phi^*}\big\} = \sum_{M,N} \frac{u^M}{M!}\frac{v^N}{N!} \int \frac{d\Phi d\Phi^*}{2\pi i} e^{-\frac{1}{y}\Phi\Phi^*} e^{M\Phi}e^{N\Phi} =$$

$$= y\sum_{M,N} \frac{e^{yM\cdot N}}{M!N!} u^M v^N, \quad (25)$$

it results into

$$Z(u,v) = \prod_{x_k} \left(y \sum_{M,N\geqslant 0} \frac{e^{yM\cdot N}}{M!N!} \big[a^2u\rho_A^\perp(x_k-b)\big]^M \big[a^2v\rho_B^\perp(x_k)\big]^N \right) =$$

$$= \exp\left\{ \sum_{x_k} \ln\big(y \sum_{M,N\geqslant 0} \frac{e^{yM\cdot N}}{M!N!} \big[a^2u\rho_A^\perp(x_k-b)\big]^M \big[a^2v\rho_B^\perp(x_k)\big]^N \big) \right\}. \quad (26)$$

The densities are slowly varying at the size a, which allows to replace the sum over x_k with the integral, $\sum_k \to 1/a^2 \int d^2x$,

$$Z(u,v) \;=\; e^C e^{W_y(u,v)}, \quad C = \ln y \frac{1}{a^2}\int d^2x, \quad z_y = 1\frac{1}{2}\frac{\sigma_{NN}^{\rm tot}}{a^2}, \quad (27)$$

$$W_y(u,v) \;=\; \frac{1}{a^2}\int d^2x \ln\big(\sum_{M,N} \frac{z_y^{M\cdot N}}{M!N!} \big[a^2u\rho_A^\perp(x-b)\big]^M \big[a^2v\rho_B^\perp(x)\big]^N\big). \quad (28)$$

The factor e^C in front is irrelevant due to Eq. (16). As $W_y(0,0) = 0$ one can set $Z(0,0) = 1$.

Now we are going to apply the same method to evaluate $J_{AB}(b)$ function (8). It is the product of two structures like (2), that is why the discrete analog of the formula (12) comprises the product of two (22) integrals,

$$J_{AB}(b) = \prod_{x_n} \int \frac{d\Phi(x_n)d\Phi^*(x_n)}{2\pi i} \prod_{y_n} \int \frac{d\Psi(y_n)d\Psi^*(y_n)}{2\pi i} \times$$

$$\times \exp\left\{ -\sum_n \frac{1}{y}\Phi(x_n)\Phi^*(x_n) - \sum_n \frac{1}{y}\Psi(y_n)\Psi^*(y_n) \right\} \times$$

$$\times \langle A| \prod_i e^{\Phi(x_i)} |A\rangle \langle A| \prod_i e^{\Psi(y_i)} |A\rangle \sum_{B'} \langle B| \prod_i e^{\Phi^*(x'_i)} |B'\rangle \langle B'| \prod_i e^{\Psi^*(y'_i)} |B\rangle. \quad (29)$$

Recalling the completeness (6) one gets

$$J_{AB}(b) = \prod_{x_n} \int \frac{d\Phi(x_n)d\Phi^*(x_n)}{2\pi i} \prod_{y_n} \int \frac{d\Psi(y_n)d\Psi^*(y_n)}{2\pi i} \times$$

$$\times \exp\left\{ -\sum_n \frac{1}{y}\Phi(x_n)\Phi^*(x_n) - \sum_n \frac{1}{y}\Psi(y_n)\Psi^*(y_n) \right\} \times$$

$$\times \; \langle A| e^{\sum_i \Phi(x_i)} |A\rangle \langle A| e^{\sum_i \Psi(y_i)} |A\rangle \langle B| e^{\sum_i \left(\Phi^*(x'_i) + \Psi^*(y'_i)\right)} |B\rangle. \quad (30)$$

Using the fact that

$$\langle A|e^{\sum_i \Phi(x_i)}|A\rangle = \left[\int d^2x \rho_A^\perp(x-b)e^{\Phi(x)}\right]^A, \quad \langle A|e^{\sum_i \Psi(y_i)}|A\rangle = \left[\int d^2y \rho_A^\perp(y-b)e^{\Psi(y)}\right]^A,$$
$$\langle B|e^{\sum_i \left(\Phi^*(x_i')+\Psi^*(y_i')\right)}|B\rangle = \left[\int d^2x \rho_B^\perp(x)e^{\Phi^*(x)+\Psi^*(x)}\right]^B,$$

and passing to the generating function we get the products of the independent integrals at the points x_i,

$$Z_J(u_A,v_A,v_B) = \prod_{x_i}\int \frac{d\Phi(x_i)d\Phi^*(x_i)}{2\pi i}\int \frac{d\Psi(x_i)d\Psi^*(x_i)}{2\pi i}\times$$
$$\times \exp\{-\frac{1}{y}\Phi(x_i)\Phi^*(x_i) - \frac{1}{y}\Psi(x_i)\Psi^*(x_i) +$$
$$+\ u_A a^2\rho_A^\perp(x_i-b)e^{\Phi(x_i)} + v_A a^2\rho_A^\perp(x_i-b)e^{\Psi(x_i)} + v_B a^2\rho_B^\perp(x_i)e^{\Phi^*(x_i)+\Psi^*(x_i)}\}.$$

Using again the identity (25) to evaluate the integrals over $\Phi(x_i)$, $\Psi(x_i)$, we arrive at the expression

$$Z(u_A,v_A,v_B) =$$
$$= \prod_{x_k}\left(y^2\sum_{M,N,K\geqslant 0}\frac{e^{y(M+N)\cdot K}}{M!N!K!}\left[a^2u_A\rho_A^\perp(x_k-b)\right]^M\left[a^2v_A\rho_A^\perp(x_k-b)\right]^N\left[a^2v_B\rho_B^\perp(x_k)\right]^K\right) =$$
$$= \prod_{x_k}\left(y^2\sum_{L,K\geqslant 0}\frac{e^{yL\cdot K}}{L!K!}\left[a^2(u_A+v_A)\rho_A^\perp(x_k-b)\right]^L\left[a^2v_B\rho_B^\perp(x_k)\right]^K,\right), \quad (31)$$

giving the generating function

$$Z_J(u_A,v_A,v_B) = e^{W_J(u_A,v_A,v_B)}, \quad (32)$$
$$W_J(u_A,v_A,v_B) = \frac{1}{a^2}\int d^2x \ln\Big(\sum_{L\leqslant 2A, K\leqslant B}\frac{z_y^{LK}}{M!N!}\left[a^2(u_A+v_A)\rho_A^\perp(x-b)\right]^L\left[a^2v_B\rho_B^\perp(x)\right]^K\Big).$$

Comparing this expression with (27) we conclude that

$$Z_J(u_A,v_A,v_B) = Z(u_A+v_A,v_B) \quad (33)$$

and, respectively,

$$J_{AB}(b) = \frac{\partial^A}{\partial u_A^A}\frac{\partial^A}{\partial v_A^A}\frac{\partial^B}{\partial v_B^B}Z(u_A+v_A,v_B)\Big|_{u_A=v_A=v_B=0},$$

or, finally

$$J_{AB}(b) = \frac{\partial^{2A}}{\partial u^{2A}}\frac{\partial^B}{\partial v^B}Z(u,v)\Big|_{u=v=0}. \quad (34)$$

4. Results of the calculations

The integrand $W(u,v)$ (28) is the logarithm of the series in u^Mv^N powers, therefore $W(u,v)$ can be expanded into u^Mv^N powers itself up to $M=2A$, $N=B$ terms. The higher orders do not contribute as is evident from Eqs. (16) and (34). After this the function $W(u,v)$ goes as the series built of the densities overlaps,

$$t_{m,n}(b) = \frac{1}{a^2}\int d^2x \left[a^2\rho_A^{\perp}(x-b)\right]^m \left[a^2\rho_B^{\perp}(x)\right]^n, \tag{35}$$

with $m \leqslant 2A$ and $n \leqslant B$. For the following calculations the nucleon density has been taken in a simple Gaussian parameterization well suited for light nuclei:

$$\rho(r) = \rho_0 e^{-\frac{r^2}{a_c^2}}. \tag{36}$$

The reason to chose this form is that it includes only one parameter, a_c, which can be directly found from the measured cross section. The value a_c is related to the mean square nuclear radius, $a_c = \sqrt{(}2/3)R_{ms}$.

The total nucleon–nucleon cross section and the slope value (averaged over pp and pn interaction) are taken in the amplitude (3) as [5, 9]

$$\sigma_{NN}^{\text{tot}} = 43 \text{ mb}, \quad \beta = 0.2 \text{ fm}^2 \tag{37}$$

for the energy around 1000 MeV per projectile nucleon.

The calculations have been carried out for the beam nuclei ^{4}He, ^{6}He, ^{8}He, ^{6}Li, ^{7}Li, ^{8}Li, ^{9}Li, ^{11}Li, ^{9}Be, ^{10}Be, ^{11}Be, ^{12}C, ^{14}C, ^{15}C scattering on ^{12}C target and for the beam nuclei ^{4}He, ^{6}He, ^{8}He, ^{8}Li, ^{9}Li, ^{9}Be, ^{10}Be, ^{11}Be scattering on ^{27}Al target.

Firstly, the ^{12}C density has been determined from the data on ^{12}C–^{12}C scattering. Using it the parameters a_c and, consequently, R_{ms} have been found for other beam nuclei scattering on ^{12}C target. The results are collected in Table 1. The mean square radius of ^{27}Al has been defined as the average values obtained from ^{4}He and ^{9}Be scattering on ^{27}Al target, the beams' radii being taken from the previous calculation for ^{12}C target (Table 2). The obtained values read

$$R_{ms}(^{12}\text{C}) = 2.52 \text{ fm}, \quad a_c(^{12}\text{C}) = 2.06 \text{ fm}, \quad R_{ms}(^{27}\text{Al}) = 3.00 \text{ fm}, \quad a_c(^{27}\text{Al}) = 2.45 \text{ fm}. \tag{38}$$

With these parameters we again found the radii of other nuclei from the known cross sections of their scattering on ^{27}Al target. These radii are collected in Table 3. Experimental errors of all the nuclei in the table except for ^{11}Li are about 2%. The error bars are generally not only due to statistical errors but may include systematical errors as well. That is why we do not present them for the calculated radii in the table. Recall that it is the interaction cross section that has been identified with the experimental one throughout the calculations. To estimate an effect of switching between the two cross sections the last two columns in all the tables are the interaction and the reaction cross sections evaluated with the same radius from the third column.

The radii extracted from the scattering on ^{12}C or ^{27}Al targets are compatible within several percent accuracy, which can be viewed as a kind of error bar for our estimation.

There are nuclei which can be treated as a composite system of a compact core made up of N_c nucleons and a relatively large halo with N_v valence nucleons around. The density of this composite system is assumed to be the sum of core and halo densities,

$$\rho(r) = N_c\rho_c(r) + N_v\rho_v(r). \tag{39}$$

Table 1. Mean square radii R_{ms} extracted by comparing the evaluated interaction cross section for the given nucleus scattering on the ^{12}C target with the experimental cross section. These radii are in the third column. The interaction and the reaction cross sections evaluated with the found radius are presented in the fourth and the fifth columns. In case of halo nuclei, ^{6}He, ^{11}Be, ^{11}Li, ^{15}C, the halo radius R_v is shown in parenthesis, the core radius R_c is taken as that for the above nuclei, ^{4}He, ^{10}Be, ^{9}Li, ^{14}C. The experimental data are taken from the review [10].

Nucleus	Experimental cross section, mb	Mean square radius, fm	Interaction cross section, mb	Reaction cross section, mb
^{4}He	503 ± 5	1.92	503	539
^{6}He	722 ± 6	2.70 (3.82)	722	753
^{8}He	817 ± 6	2.76	817	851
^{9}Be	806 ± 9	2.59	806	845
^{10}Be	813 ± 10	2.52	813	855
^{11}Be	942 ± 8	2.94 (5.60)	942	981
^{12}Be	927 ± 18	2.77	927	968
^{6}Li	688 ± 10	2.48	688	721
^{7}Li	736 ± 6	2.54	736	771
^{8}Li	768 ± 9	2.55	768	806
^{9}Li	796 ± 6	2.55	796	835
^{11}Li	1040 ± 60	3.29 (5.5)	1040	1075
^{12}C	853 ± 6	2.52	853	898.5
^{14}C	880 ± 19	2.50	880	928
^{15}C	945 ± 10	2.64 (4.09)	945	992

Table 2. Mean square radius of ^{27}Al target extracted from the scattering of beam ^{4}He and ^{9}Be nuclei whose radii are taken from Table 1. The radii are evaluated by matching the interaction cross sections. The interaction and the reaction cross sections calculated with these radii are presented in the fourth and the fifth columns. The experimental cross sections are taken from the review [10].

Nucleus	Experimental cross section, mb	Mean square radius, fm	Interaction cross section, mb	Reaction cross section, mb
^{4}He	780 ± 13	3.00	780	828
^{9}Be	1174 ± 11	3.00	1174	1224

Both densities are taken in Gaussian form (36), with the parameters a_c being expressed through the mean square radii of the core and the halo. This is what is called Gauss–Gauss (GG) parameterization [11].

Following [11] ^{6}He nucleus is regarded as ^{4}He core plus two neutrons halo, ^{11}Be as ^{11}Be plus one neutron halo, ^{11}Li as ^{9}Li plus two neutrons halo, ^{15}C as ^{14}C plus one neutron halo.

In calculations the ^{6}He core radius, R_c, has been chosen as the radius of ^{4}He nucleus. Plugging it into the density (39) (normalized to unity), the halo radius, R_v, for ^{6}He is found by matching the interaction cross section to the experimental cross section of ^{6}He scattering on ^{12}C or ^{27}Al targets. The mean square radius of the whole composite system is evaluated as $R^2_{ms} = (R^2_c + R^2_v)/(N_c + N_v)$. The calculations for other halo nuclei have been done in the same manner.

Tables 1 and 3 show agreement of the results obtained for ^{6}He on ^{12}C and ^{27}Al targets both for the core and the halo radii, the halo radius being considerably larger than that of the

Table 3. Mean square radii R_{ms} extracted by comparing the evaluated interaction cross section for the given nucleus scattering on the ^{27}Al target with the experimental cross section. These radii are in the third column. The interaction and the reaction cross sections evaluated with the found radius are presented in the fourth and the fifth columns. In case of halo nuclei, ^{6}He, ^{11}Be, the halo radius R_v is shown in parenthesis, the core radius R_c is taken as that for the above nuclei, ^{4}He, ^{10}Be. The experimental data are taken from the review [10].

Nucleus	Experimental cross section, mb	Mean square radius, fm	Interaction cross section, mb	Reaction cross section, mb
^{6}Li	1010 ± 11	2.43	1010	1055
^{7}Li	1071 ± 7	2.50	1071	1116
^{8}Li	1147 ± 14	2.62	1147	1193
^{9}Li	1135 ± 7	2.48	1135	1187
^{10}Be	1153 ± 16	2.45	1153	1208
^{11}Be	1382 ± 25	3.04 (6.45)	1382	1432

core. The results for the core radius of ^{11}Be satisfactorily agree for both targets. The ^{11}Be halo comes out to be evidently larger than the core though a certain discrepancy between two targets needs a more detailed analysis. Among the other beam nuclei we confirm a sizable halo for ^{11}Li on ^{12}C target, which actually was the first one experimentally discovered [4], and a not very notable halo for ^{15}C (Table 1).

5. Conclusion

The reaction and the interaction cross sections have been calculated for a number of beam nuclei scattering on ^{12}C and ^{27}Al targets. By comparing the calculated values with the experimental data the mean square radii of the nuclear density distribution have been extracted. They are presented in Tables 1 and 3. The difference between the two cross sections in Tables 1 and 3 varies from 2% for He isotopes to almost zero for more heavy nuclear beams in a qualitative agreement with [2].

There is a lot of experimental data on halo nuclei, especially concerning ^{11}Li. Getting access to nucleon distributions requires, however, an additional theoretical analysis, which is model or approximation dependent. The radius of ^{11}Li halo lies in various calculations in the range 4.8–5.68 fm [12, 13]. Our result is in this interval. At the same time, the mean square ^{11}Li radius obtained here exceeds 3.05–3.20 fm values [12, 13]. It also holds for other obtained radii, they are larger than those listed in the review [10]. This is presumably because of the optical model approximation used to extract R_{ms} from the cross sections. The optical model accounts for a part of the screening effects, the rigid target approximation includes more corrections, whereas our approach captures almost all of them. High-order corrections reduce the output cross section. That is why the larger radius has to be taken to recover the same experimental input.

The results obtained for ^{12}C and ^{27}Al targets are in fairly agreement. A possible reason for differences is in oversimplified expression for the nuclear density, especially for the halo. Another reason could be in a complex structure of unstable nuclei. The nucleus ^{11}Be, for example, admits an interpretation as a two body bound (resonance) state of ^{10}Be and neutron. But a similar two body interpretation is hardly valid for $^{6}\mathrm{He} = {}^{4}\mathrm{He} + 2n$ as there is no known two-neutron bound state.

Conflicts of interest

The authors declare no conflict of interest.

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